Engineering Knowledge Transfer Units to Increase Student´s Employability and Regional Development

Teaching “Basics in Vehicle Dynamics” 1

by Dr. Karl Reisinger

Intro, Tire, Longitudinal Dynamics

Co-funded by the Erasmus+ Programme of the European Union

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FH-Joanneum GmbH.
University of Applied Sciences, Graz, Austria

UNITED

Institute of Automotive Engineering

- Bachelor‘s Degree Program
- Master‘s Degree Program



Dr. Karl Reisinger
Assoz. Prof.(FH)

- Vehicle Dynamics
- Mechatronics

Content of Slot 1 + 2

- How do we teach Vehicle Dynamics in Bachelor's and Master's degree program of UAS Graz.
- My Presentation
  - Aim, Qualification for the courses, location in curriculum.
  - Our Content – Overview with examples
    - Tire's behaviour, Longitudinal dynamics, Lateral Dynamics, Vertical dynamics
- **Group Discussion**
  - **Presentation and Discussion.**

Questions are welcome, while the presentation

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Aim of the Vehicle Dynamics courses

Bachelor‘s Students shall know…

- **How does a car move?**
  Basic knowledge, terms, approaches
- The tire is the only contact to the road!
  - Primary spring
  - No Force w/o Slip
  - Nonlinear behaviour
  - Combined long. and lateral force
- Longitudinally
  - drag resistances, modelling
  - Longitudinal load transfer
  - Engine power, speed, for accelerated motion, energy consumption
  - Forward, backward simulation models

Master‘s shall know …

- **How to make a car faster and saver?**
  Bachelor’s knowledge but deeper
- Tire behaviour under combined load and models
  - TM-Easy by Hirschberg-Rill
  - Pacejka’s approach in principle

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Aim of the Vehicle Dynamics courses 2

Bachelor‘s Students shall know…

- **How does a car move?**

  Basic knowledge, terms, approaches
- Laterally
  - The equations to get Single Track Model’s ODE’s with const. speed.
  - Tests to get transient and steady state behaviour.
  - Under-/Oversteering
- Vertically
  - Parameters for Comfort
  - Describe road roughness

Master‘s shall know …

- **How to make a car faster and saver?**

  Bachelor’s knowledge but deeper
- Laterally
  - Lateral load transfer
  - Drive the fastest lap
  - G-g-diagram and Milliken-Moments-Diagram
- Ride – Vertical Dynamics
  - Find the optimal suspension spring and damper in terms of comfort, driving safety and aerodynamics.

Aim of the Vehicle Dynamics courses 3

Bachelor‘s Students shall know…

- **How does a car move?**

  Basic knowledge, terms, approaches

- Simulation Methods
  - Backwards Simulation (Matlab)
  - Forward Simulation (Simulink)
  - What commercial programs deliver, number of it’s parameters (veDYNA/TESIS)

Master‘s shall know …

- **How to make a car faster and saver?**

  Bachelor’s knowledge but deeper

- Simulation Methods
  - Lap time using 1 DOF model and g-g-diagram (Matlab)
  - Using AVL/VSM: parameter identification using measured data, lap time sim, sensitivity analysis, energy optimization

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Qualification for the courses

Bachelor‘s Program

- 5th semester, 2 ECTS
  - 20h Lecture, 4 Practices
- Prior Courses necessary
  - Engineering Mathematics
  - Basics in Mechanics/Dynamics
  - Characteristics of Electric Drives
  - Matlab/Simulink
  - Internal Combustion Engines
- In Parallel to this course
  - Chassis Engineering
  - Drive and Propulsion Technology
  - In vehicle testing

Master‘s Program

- 3rd semester, 2 ECTS
  - 20h Lecture, 4 Practices
- Prior Courses necessary
  - Our Bachelor‘s program or
  - Bachelor in Mechanical Engineering, Mechatronic Engineering
    + Supplementary Exams in
    - Matlab, Simulink,
    - Vehicle Dynamics & Chassis Eng.

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Overview to Tire's behaviour

How to understand slip, tire's nonlinearity and its effects.

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What drives the car?

- The tire is the only part transferring forces to
  - accelerate/brake
  - drive a turn

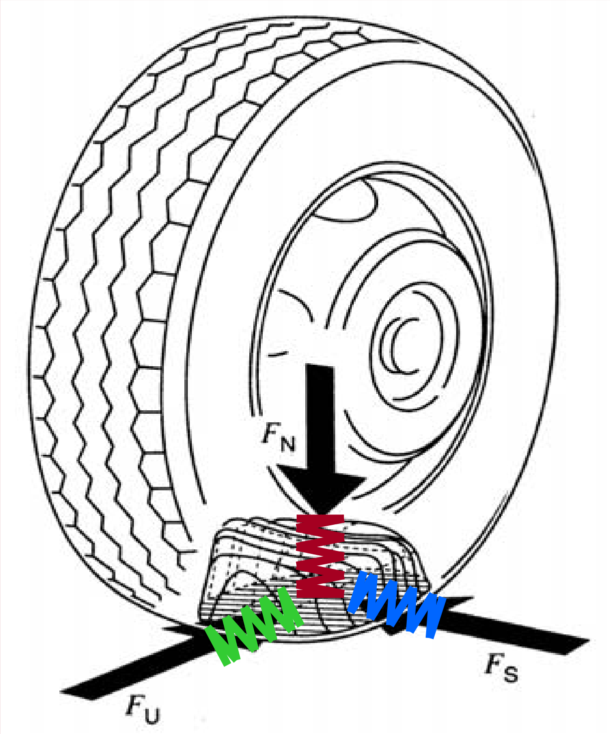
Tyre Production:
http://www.youtube.com/watch?v=Li-MKobBg5w

Bias-Ply Tyre vs. Radial Tyre
http://www.youtube.com/watch?v=Ii0n8SK9V2s

Picture: K. Reisinger

Tire = Primary Spring

- … in 3 directions (3 DOF)
- **comfort**
  - filters vibrations coming from the road
- **traction**
  - less motion of suspension
  - less wheel load variance
- **rolling efficiency**
  - equalizes small unevenness’



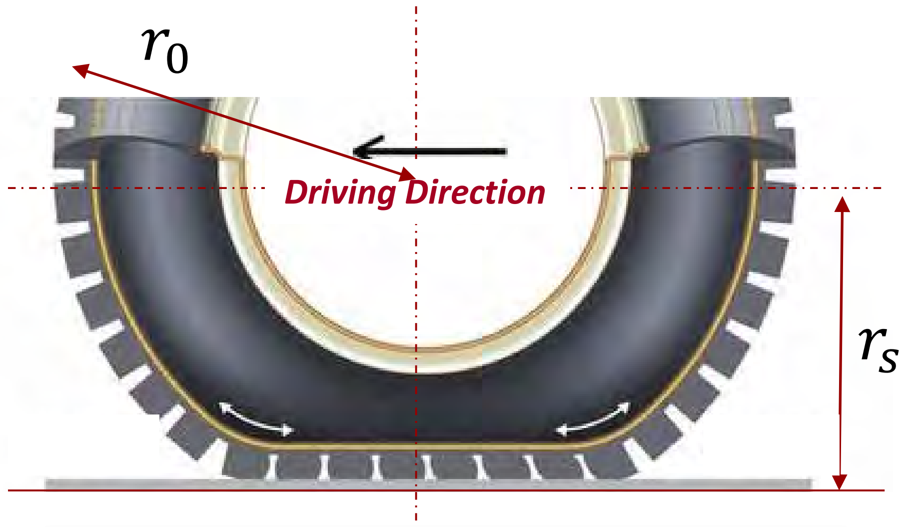
Wheel with pressure distribution in contact area [Bosch02]

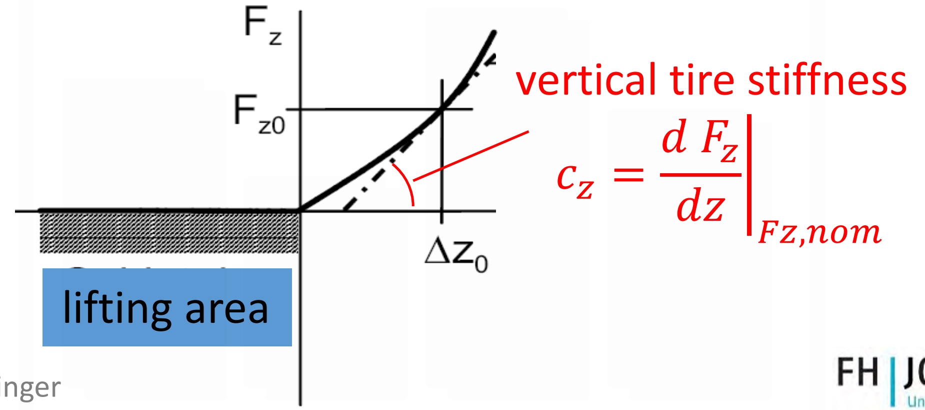
Air-filled tires n vs. pure rubber tires
http://www.youtube.com/watch?v=jsxP7SYRF60&feature=related

Tire Radii

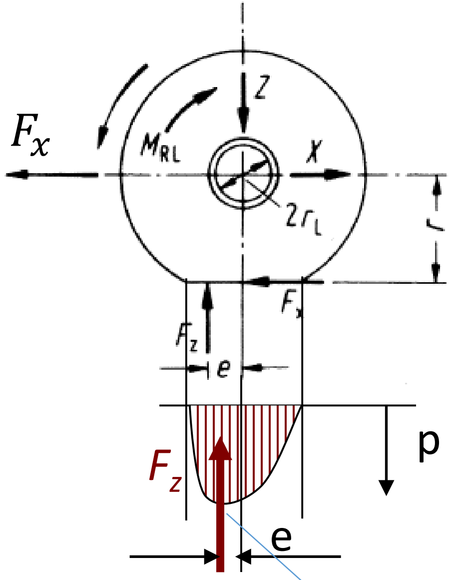
- Outer Radius r_0
- Static Loaded Radius r_s
- $r_s = f(F_z, p)$
- Effective Radius $r_e = \frac{U_{eff}}{2\pi}$
- $r_e = f(p, v)$
- Estimations[1)] $\mathrm{r_e} \approx \frac{2\, r_0 + r_s}{3}$

1)
- Reimpel, Grundlagen der Fahrwerktechnik, Vogel 2000
- Rill G.: Road Vehicle Dynamics, CRC Press, 2011





Rolling Resistance Coefficient f_R



Distribution of vertical load if rolling to left side

$$F_x \cdot r_e = F_z \cdot e$$
$$F_x = f_R \cdot F_z$$

- f_R depends on
  - tire radius
  - toe in, camber
  - pressure
  - road
    asphalt, roughness, earth, sand, …
  - speed
  - (Hub & brake friction)

Efficiency Class of tires

Reifen der Klasse C1		Reifen der Klasse C2		Reifen der Klasse C3	
CR in kg/t	Energieeffi-zienzklasse	*CR* in kg/t	Energieeffi-zienzklasse	*CR* in kg/t	Energieeffi-zienzklasse
$CR \leq 6{,}5$	A	$CR \leq 5{,}5$	A	$CR \leq 4{,}0$	A
$6{,}6 \leq CR \leq 7{,}7$	B	$5{,}6 \leq CR \leq 6{,}7$	B	$4{,}1 \leq CR \leq 5{,}0$	B
$7{,}8 \leq CR \leq 9{,}0$	C	$6{,}8 \leq CR \leq 8{,}0$	C	$5{,}1 \leq CR \leq 6{,}0$	C
–	D	–	D	$6{,}1 \leq CR \leq 7{,}0$	D
$9{,}1 \leq CR \leq 10{,}5$	E	$8{,}1 \leq CR \leq 9{,}2$	E	$7{,}1 \leq CR \leq 8{,}0$	E
$10{,}6 \leq CR \leq 12{,}0$	F	$9{,}3 \leq CR \leq 10{,}5$	F	$CR \geq 8{,}1$	F
$CR \geq 12{,}1$	G	$CR \geq 10{,}6$	G	–	G

C1 .. Passenger Car, C2 .. Light Trucks, C3 .. Trucks

https://ec.europa.eu/transparency/regdoc/rep/1/2009/DE/1-2009-348-DE-F2-1.Pdf

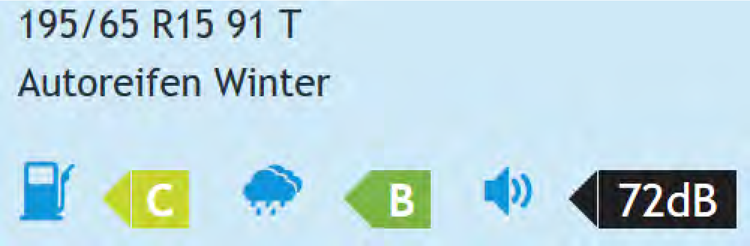
Test procedure
https://eur-lex.europa.eu/legal-content/EN/TXT/?qid=1570609195857&uri=CELEX:42011X1123(03)
plain steel drum, $d_{Drum} = 2\ m$, 25 °C, for C1 : speed=80 km/h, Fz=80% of max. tire load

Efficiency Class of tires in the future

C1 tyres		C2 tyres		C3 tyres	
RRC in kg/t	*Energy efficiency class*	*RRC in kg/t*	*Energy efficiency class*	*RRC in kg/t*	*Energy efficiency class*
$RRC \leq 5,4$	*A*	$RRC \leq 4,4$	*A*	$RRC \leq 3,1$	*A*
$5,5 \leq RRC \leq 6,5$	*B*	$4,5 \leq RRC \leq 5,5$	*B*	$3,2 \leq RRC \leq 4,0$	*B*
$6.6 \leq RRC \leq 7.7$	*C*	$5.6 \leq RRC \leq 6.7$	*C*	$4.1 \leq RRC \leq 5.0$	*C*
$7,8 \leq RRC \leq 9,0$	*D*	$6,8 \leq RRC \leq 8,0$	*D*	$5,1 \leq RRC \leq 6,0$	*D*
$9,1 \leq RRC \leq 10,5$	*E*	$8,1 \leq RRC \leq 9,2$	*E*	$6,1 \leq RRC \leq 7,0$	*E*
$RRC \geq 10,6$	*F*	$RRC \geq 9,3$	*F*	$RRC \geq 7,1$	*F*

C1 .. Passenger Car, C2 .. Light Trucks, C3 .. Trucks

https://ec.europa.eu/transparency/regdoc/rep/1/2018/EN/COM-2018-296-F1-EN-ANNEX-1-PART-1.PDF (2018)



Comparison Sports to ECO-Tires, an estimation

- Vehicle Mass $\mathrm{m_{veh}} = 1600\ \mathrm{kg}$
- neglect lift force
- Lifetime L=44 000 km1)
  - 1 Litre petrol costs 1.20 €, fuel value $H_u = 11.5\,\frac{\mathrm{kWh}}{\mathrm{kg}} \cdot 0.75\,\frac{\mathrm{kg}}{\mathrm{l}}$, gives 2.32 kg CO2
  - Mean efficiency of Spark Ignited Engine : approx. 25% in cycle

- Energy saves in kWh if you use Class B or Class E per 100 km
- How does consumption sink, l/100km?
- CO2 saves in g/km?
- Safed money while lifetime?

1) https://www.reifendirekt.at/FAQs/Fragen_rund_um_die_Produkte_Fragen_zu_Reifen_Fragen_zum_Reifenalter.html#Fragen_rund_um_die_Produkte_Fragen_zu_Reifen_Fragen_zum_Reifenalter-246

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Flatbed Tire Testing Machine

Tire in Testing Machine
http://www.youtube.com/watch?v=W8UiE7yvO_M&feature=related

https://www.youtube.com/watch?v=dZhTdljr2Zc&feature=related

Tire testing under real road conditions

- Real road condition
- Results change with weather
- In car measurement
  - inclination angle is not well defined
- Measurement-Trailer, Measurement-Truck

Tire Measurement Trailer

Measurement Truck: http://www.fkfs.de

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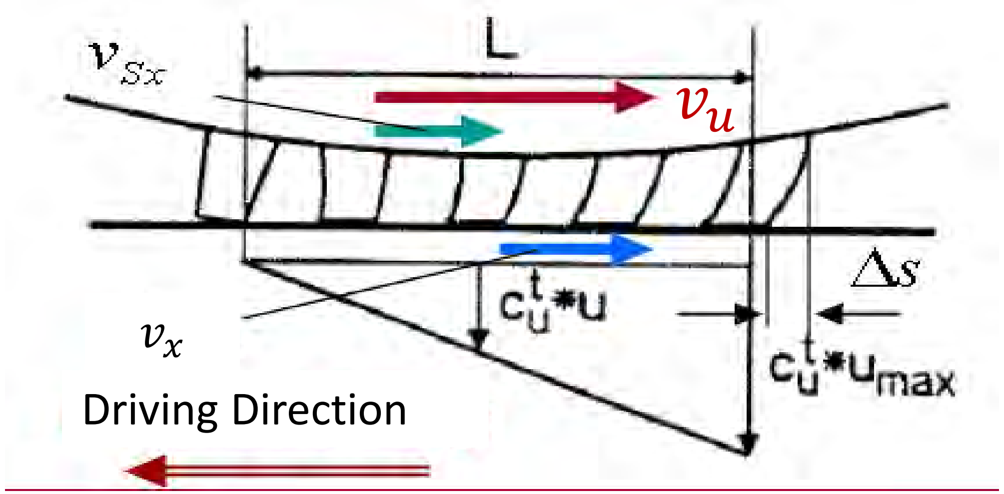
Brush Model

- **No stress without strain → No tire force without slip!**
- Driven wheels turn fasten than non-driven
- Braked wheels turn slower
- Brush model
  - treat element **front** position: **unloaded and undeformed**
  - rubber deforms due to load within the print front to rear
  - circumference speed $v_u = r_e \cdot \omega$
  - Different to the speed of the wheel centre.
  - While wheel centre moves print length L, the tire has to move by Δs more to „load“ the rubber.
- NO SLIP – NO GRIP

Example: Wheel with drive torque, not braking



Rel. velocities seen from wheel centre

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Tire Slip

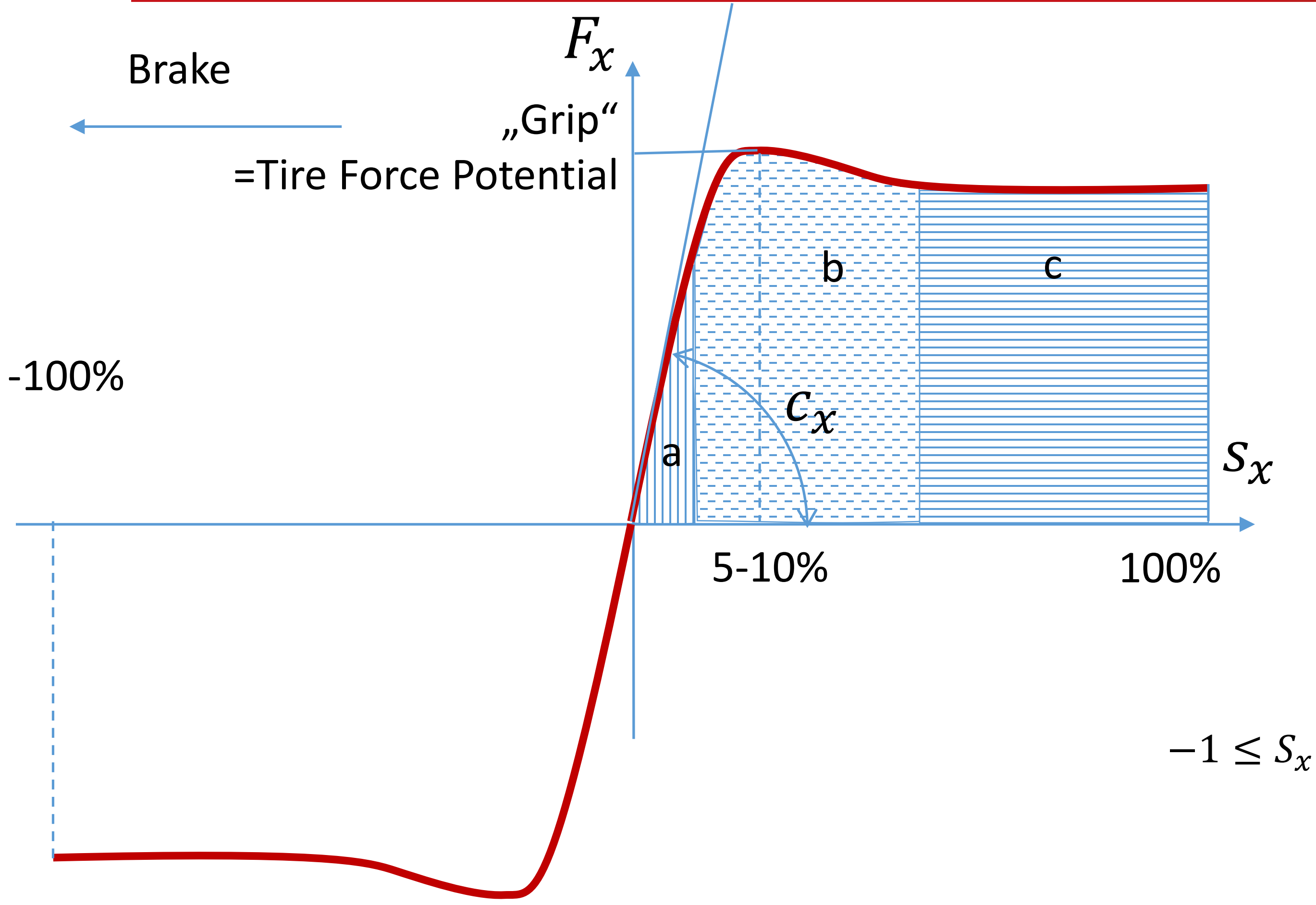
$$r_e \cdot \omega = v_{wheel} + v_{sx}$$

- v_x… rim centre velocity over ground
  - $v_x = v_{veh}$ without steering, driving straight forward
- v_{Sx}… Slip velocity

$$-1 \leq S_x \leq 1, S_x = \begin{cases} \dfrac{r_e \cdot \omega - v_x}{r_e \cdot \omega} & \ldots drive\ mode \\ 0 & \ldots rolling\ with\ rolling\ resistance \\ \dfrac{r_e \cdot \omega - v_x}{v_x} & \ldots thrust\ mode\ or\ braking \end{cases} = \frac{r_e \cdot \omega - v_x}{\max(r_e \cdot \omega, v_x, eps)}$$

- S_x… Slip according Mitschke/Wallentowitz
- eps… small number to avoid div. by zero at stillstand.

Force-Slip-Characteristics of a tire



- F_x … longitudinal force transferred in print
- s_x … longitudinal slip
- Rolling: $s_x = 0$ by definition of r_e
- $c_x = \frac{dF_x}{ds}$ … Long. Tire slip stiffness for lin. models
- $\mu_x(s_x) = \frac{F_x}{F_z}$ … tire force coefficient
- Mitschke/Wallentowitz:

$$-1 \leq S_x \leq 1, S_x = \begin{cases} \dfrac{r_e \cdot \omega - v_x}{r_e \cdot \omega} & \ldots thrust\ mode \\ 0 & \ldots rolling\ with\ rolling\ resistance \\ \dfrac{r_e \cdot \omega - v_x}{v_x} & \ldots coast\ down\ or\ braking \end{cases}$$

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Lateral Slip =Side Slip

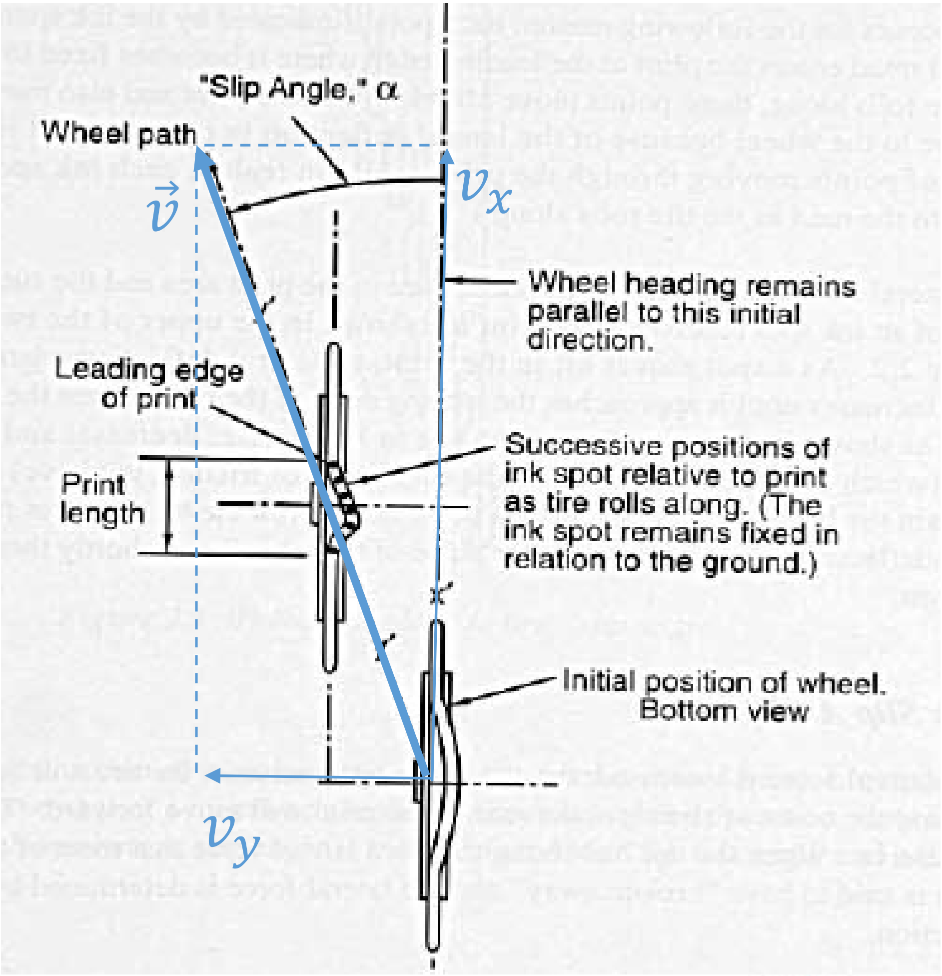
- Side Slip

$$s_y = \frac{v_y}{v_x}$$

- Side Slip Angle α

$$\tan(\alpha) = s_y$$

- Self alignment torque
  ->the lateral force distribution



GM Wheel [Milliken95]

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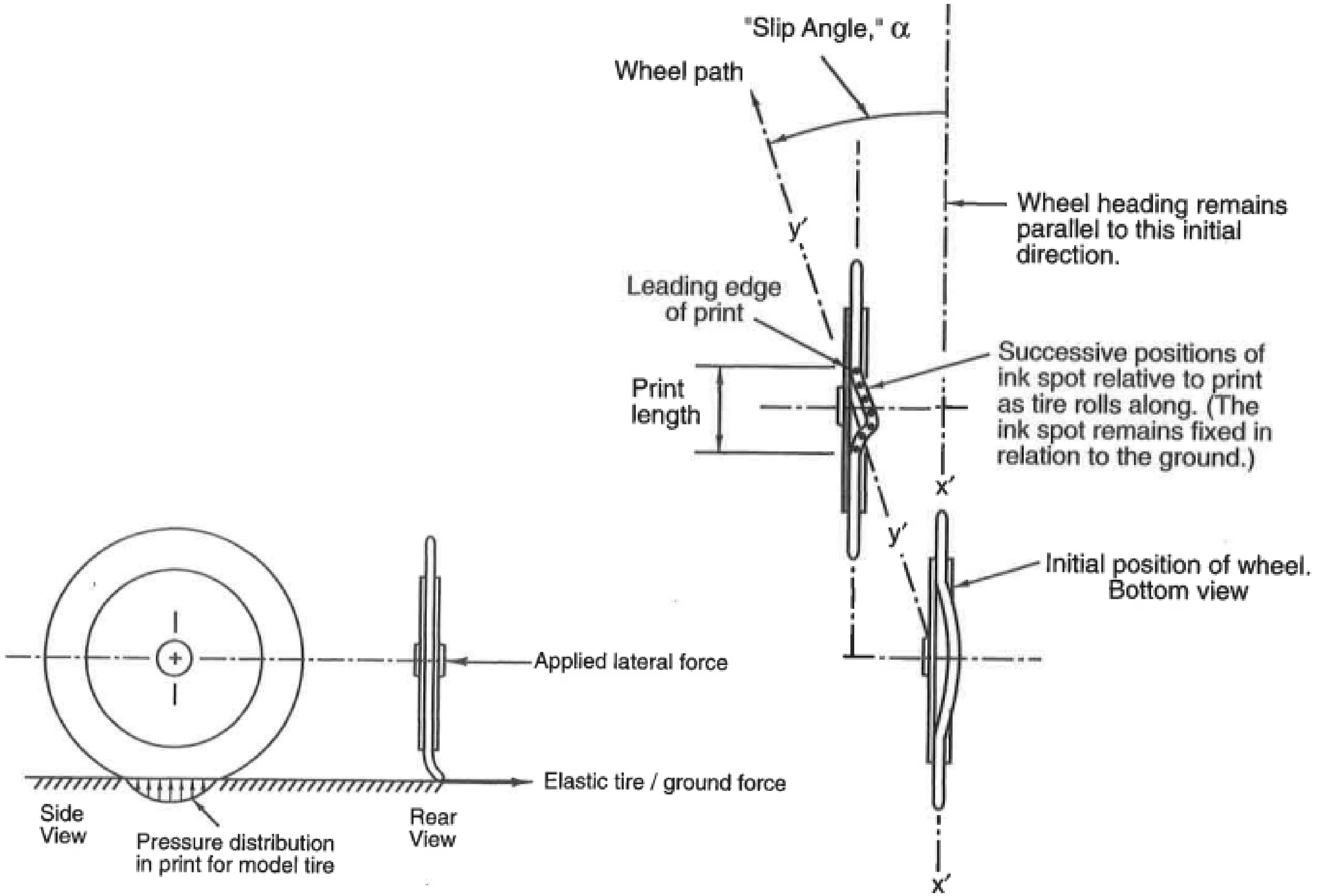
Tyre Slip in x and y

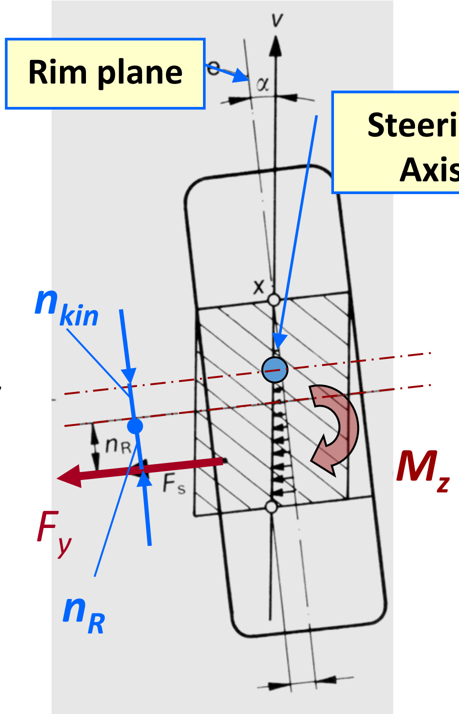
- no stress without strain
  - no force transfer without deformation
- deformation of a rolling wheel makes wheel slip
  - regular slip is not skidding of rubber on road surface!

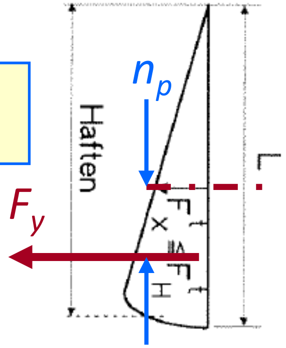


[Milliken W., Milliken D.: Race Car Vehicle Dynamics, SAE 1995]

Self Aligning Torque M_z due to Lat. Force

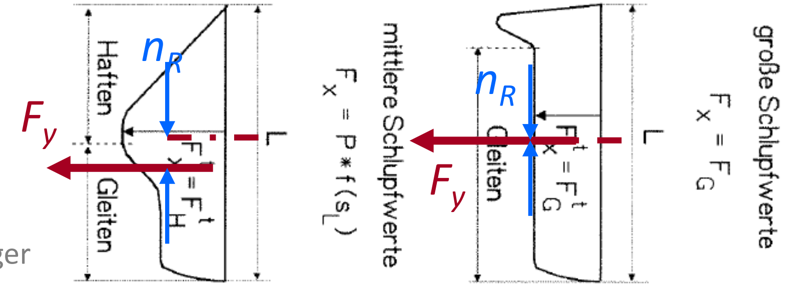
- $M_z = F_y \cdot (n_p + n_{Kin})$
  - $F_y = \int_{x=0}^{L} \frac{dF_y}{dx}\,dx$ is not symmetrically distributed!
- tire trail, (pneumatic) trail n_p
  - $n_R \cong \frac{1}{6} \cdot L$, L … print length al t low F_y
  - n_p decreases if the max. friction potential was reached in areas of the print
  - M_z is a good feedback to the driver for road friction (less if slippery)
- kinematic trail n_{Kin}
  - due to steering geometry
  - given by the engineer





small side force
- Print sticks totally on road
- Linear distribution
- $n_p \cong \frac{L}{6}$

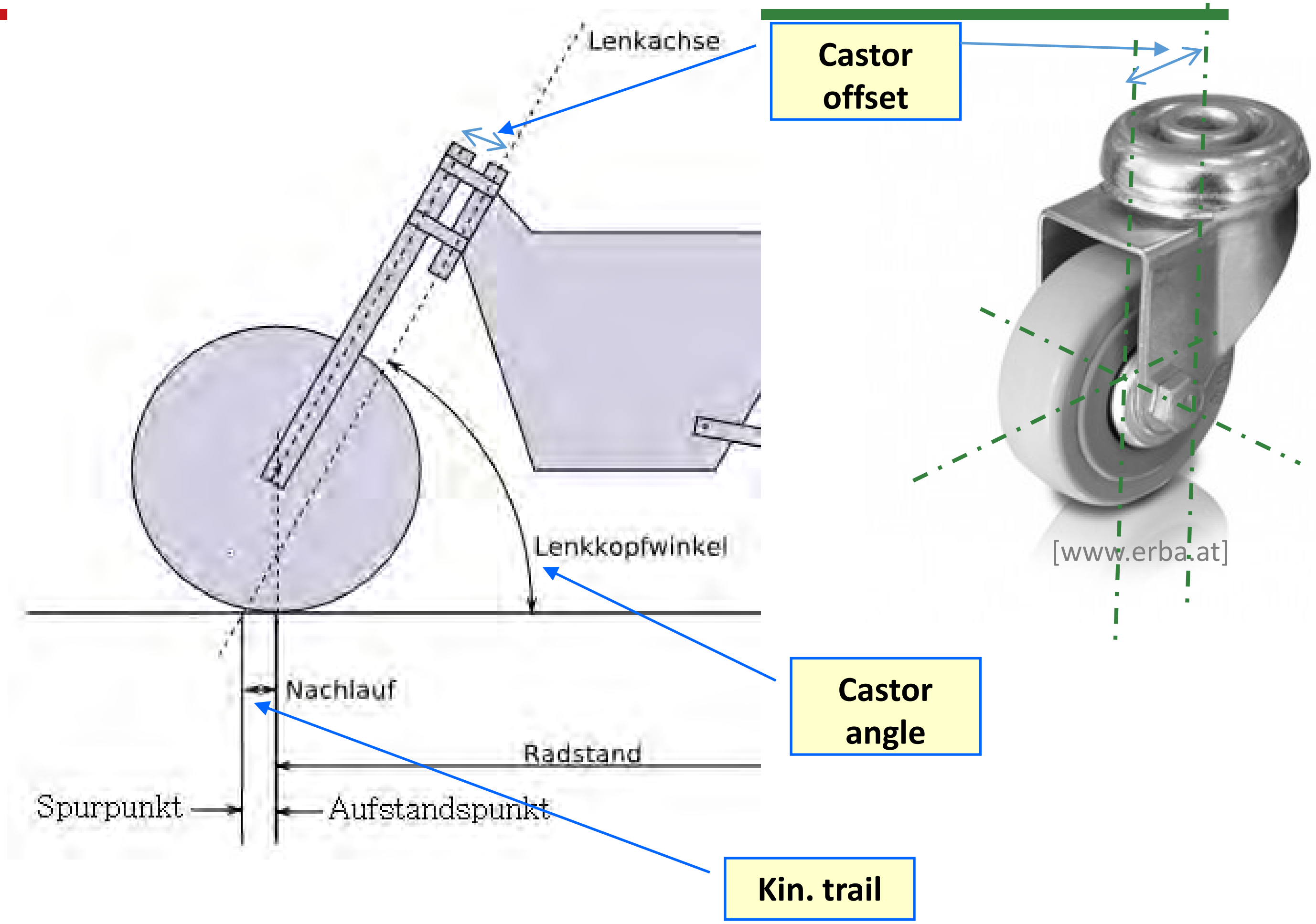
high side force
- Tire sticks within the front area, slides after reaching a maximum
- Squeezing
- Resultant side force moves forward



Geometric trail

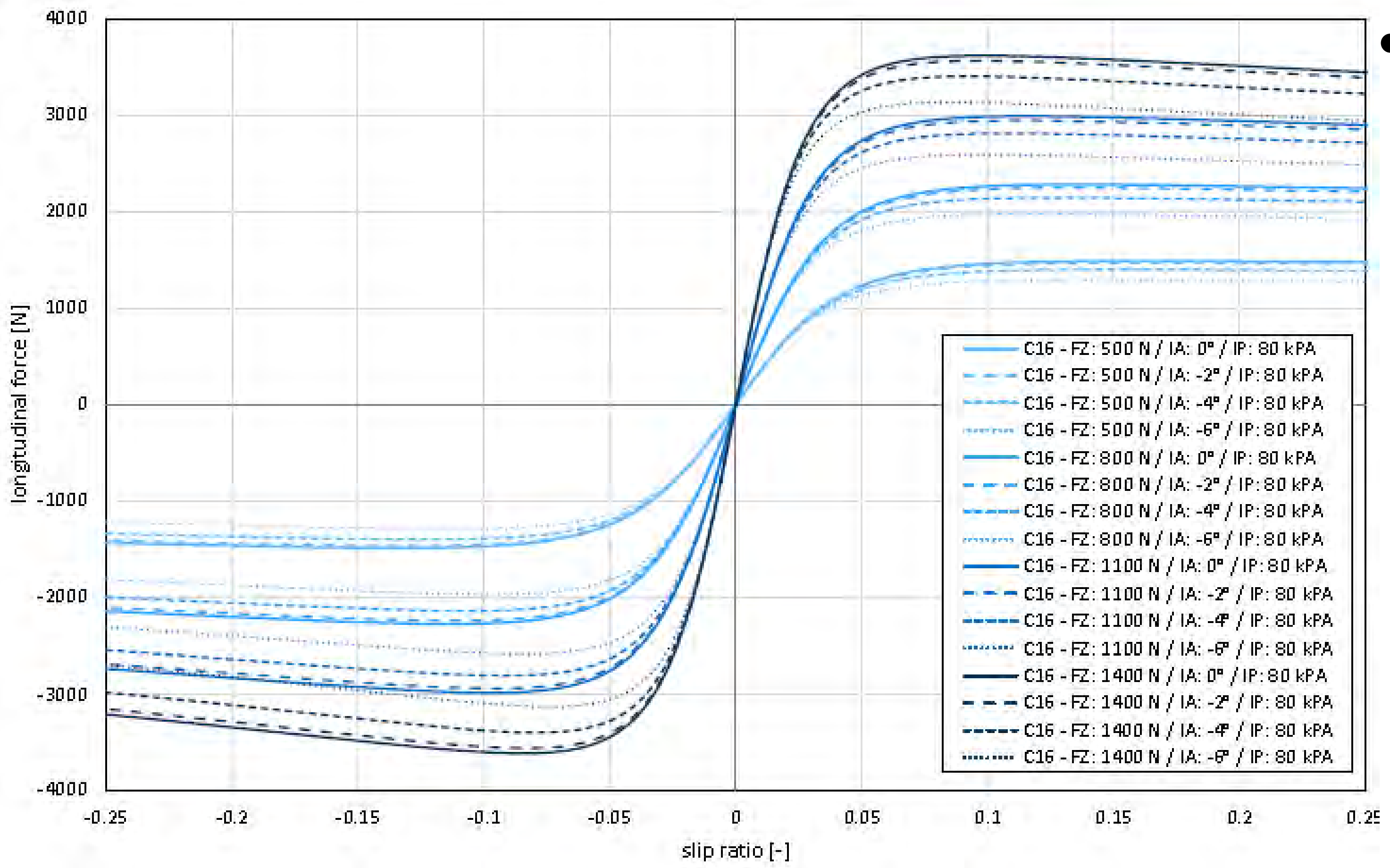
- = kinematic trail, mechanical trail, Caster(Am.),Castor(Brit.)
  - Distance between wheel centre projected to road to intersection of steering axis and road
  - Given by
    - Caster offset
    - Caster angle

$$n = n_R + n_G$$



https://de.wikipedia.org/wiki/Nachlauf_%28Lenkung%29#/media/Datei:Lenkgeometrie_Zweirad.png

Wheel Load Dependence



- Nonlinear!
  - If you double the load, you get less than double force.

  $F_x(2 \cdot F_z) < 2 \cdot F_x(F_z)$

  $\mu_x(2 \cdot F_z) < \mu\_x(F_z)$

Same in lateral direction!

Source: Conti Formula S racing tire

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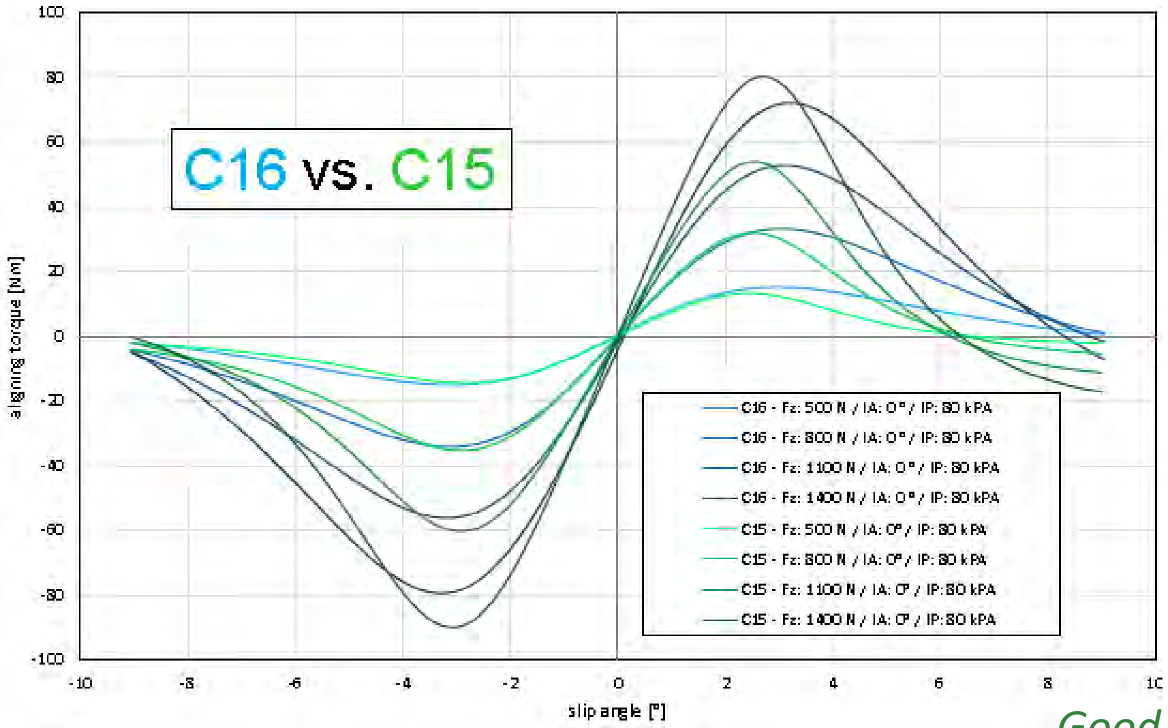
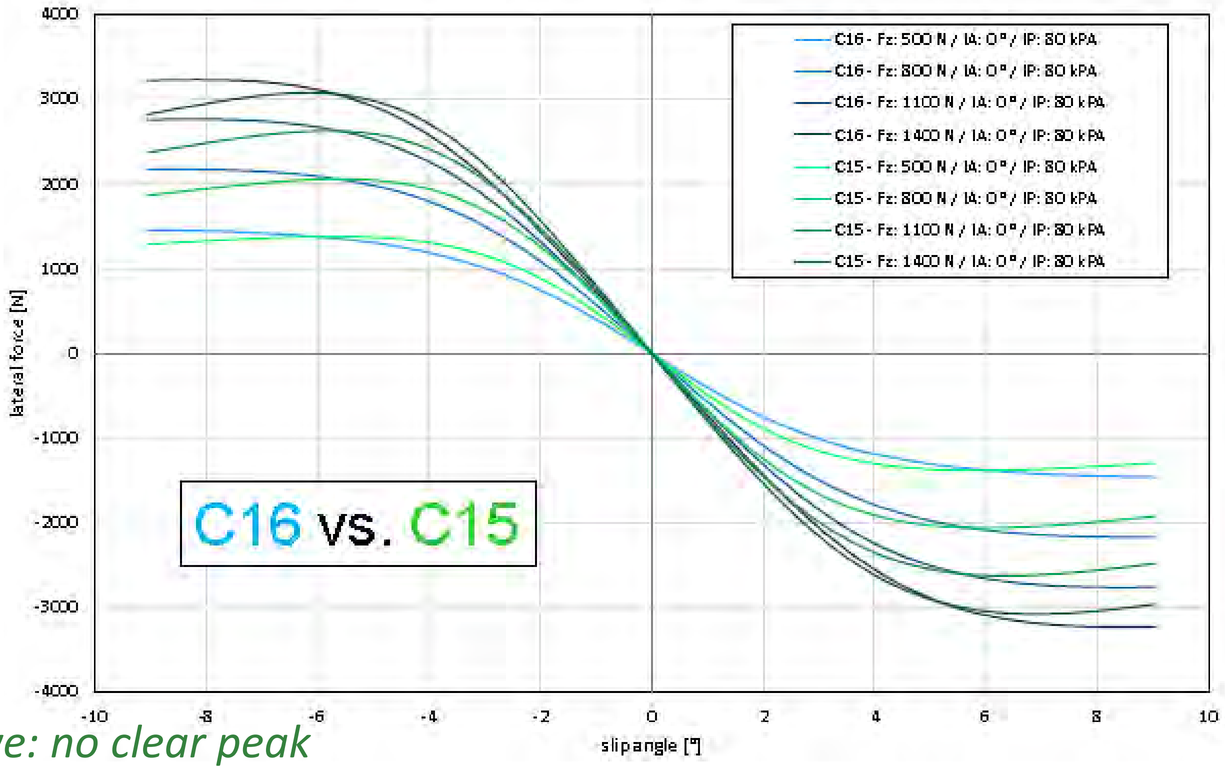
Self alignment torque and lateral force





Source: Continental
C15, C16 2 types of Formula S Racing tires

Good to drive: no clear peak
Self Alignment Torque = Feedback to driver about grip μ_{max} !
Nonlinearity about Fz:
„You loose more at inner side, than you gain in outer side“

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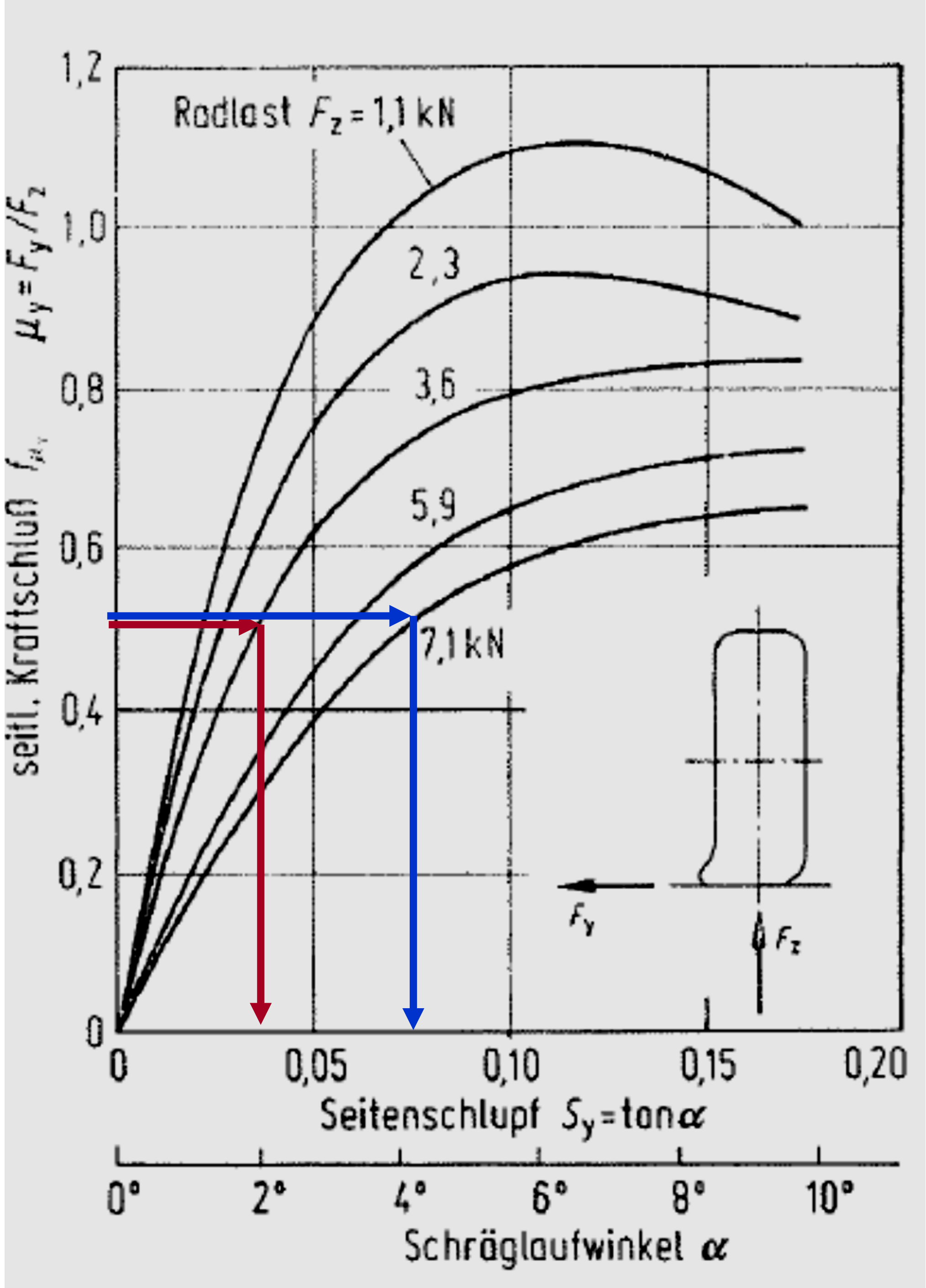
Influence of wheel load distribution left right

- Example
  - 1st axle: same load left and right
    - Roll torque is transferred by 2nd axle
      Fz1=Fz2=3600N,
      Fy1=Fy2=1800N,
      μy=Fy/Fz=0.5
      →α=2°
  - 2nd axle: inner wheel is nearly lifted
    - Fz1=7200N, Fz2=0 N
      Fy1=3600N, Fy2=0
      μy1=Fy1/Fz1=0.5
      →α=4°
- The axle with high wheel load difference has more side slip.
  - Influenced by Anti Roll Bar.



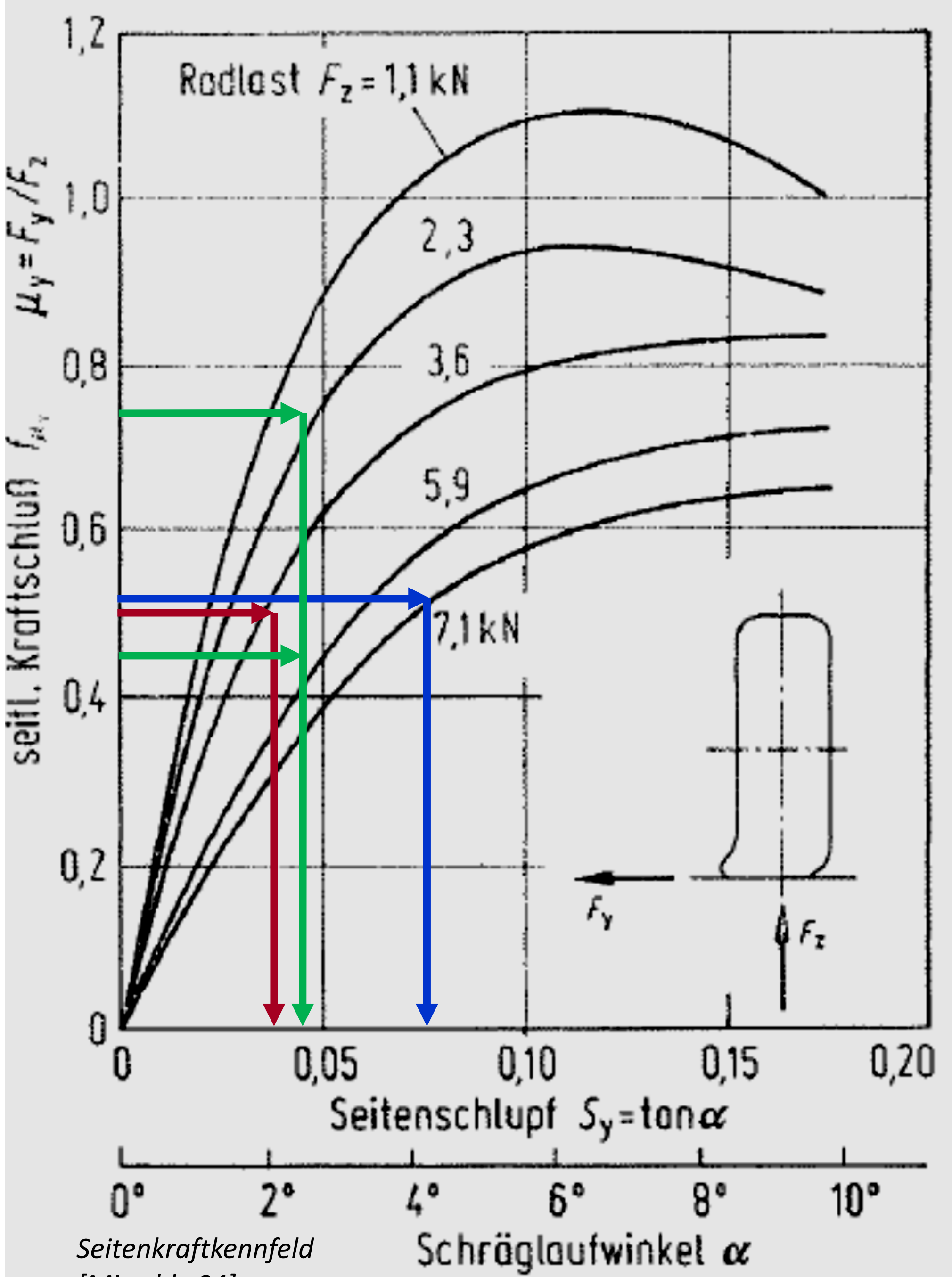
Seitenkraftkennfeld [Mitschke04]

Influence of wheel load distribution left right

Other Suspension:

Roll moment is transferred 50% / 50% front and rear axle

- Front and rear axle are loaded equally.
- $F_{z,i} = F_{z,stat} \pm \Delta F_z$
- *VA und HA:* F_{z1}=5400 N, F_{z2}=1800 N
- *Sum of lat. tire forces at one axle is given*
  $m_{Axle} \cdot a_y = F_{y1} + F_{y2} = 3600\ \mathrm{N}$
- *Same tire slip angle left and right, because the wheels are connected by the car (exact at wide curves)*
- *We search the slip, where* $F_{y1} + F_{y2} = 3600\ \mathrm{N}$
- F_{y1}=0.42*5400, F_{y2}=0.73*1800
  → α=2.5°



Seitenkraftkennfeld [Mitschke04]

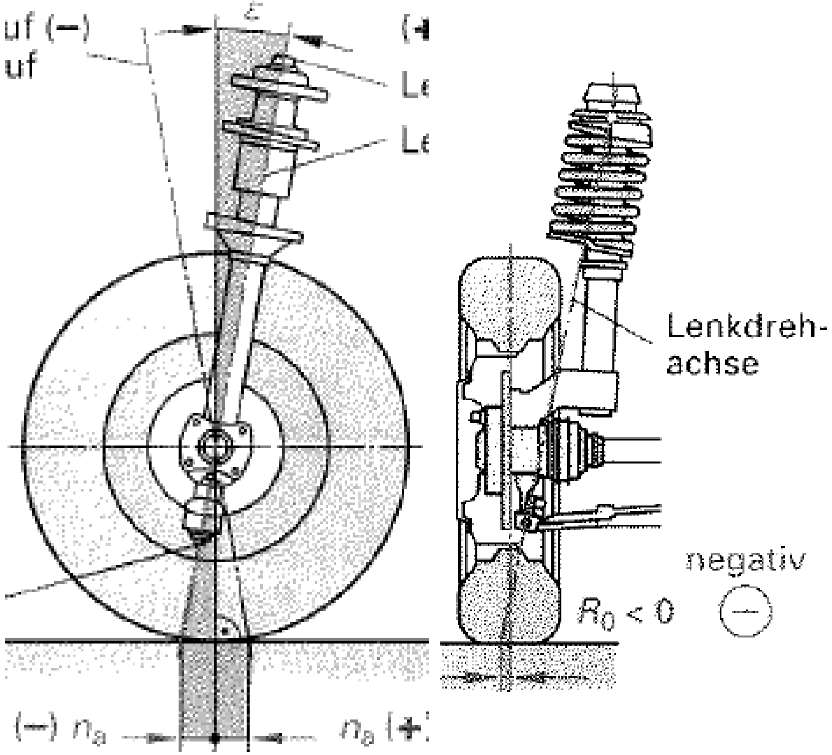
Cause of Wheel Steering Moment $\boldsymbol{M_z}$

From tire side force

- castor angle
- castor offset

→ Kinematic Trail

- pneumatic trail



[Reimpel00]

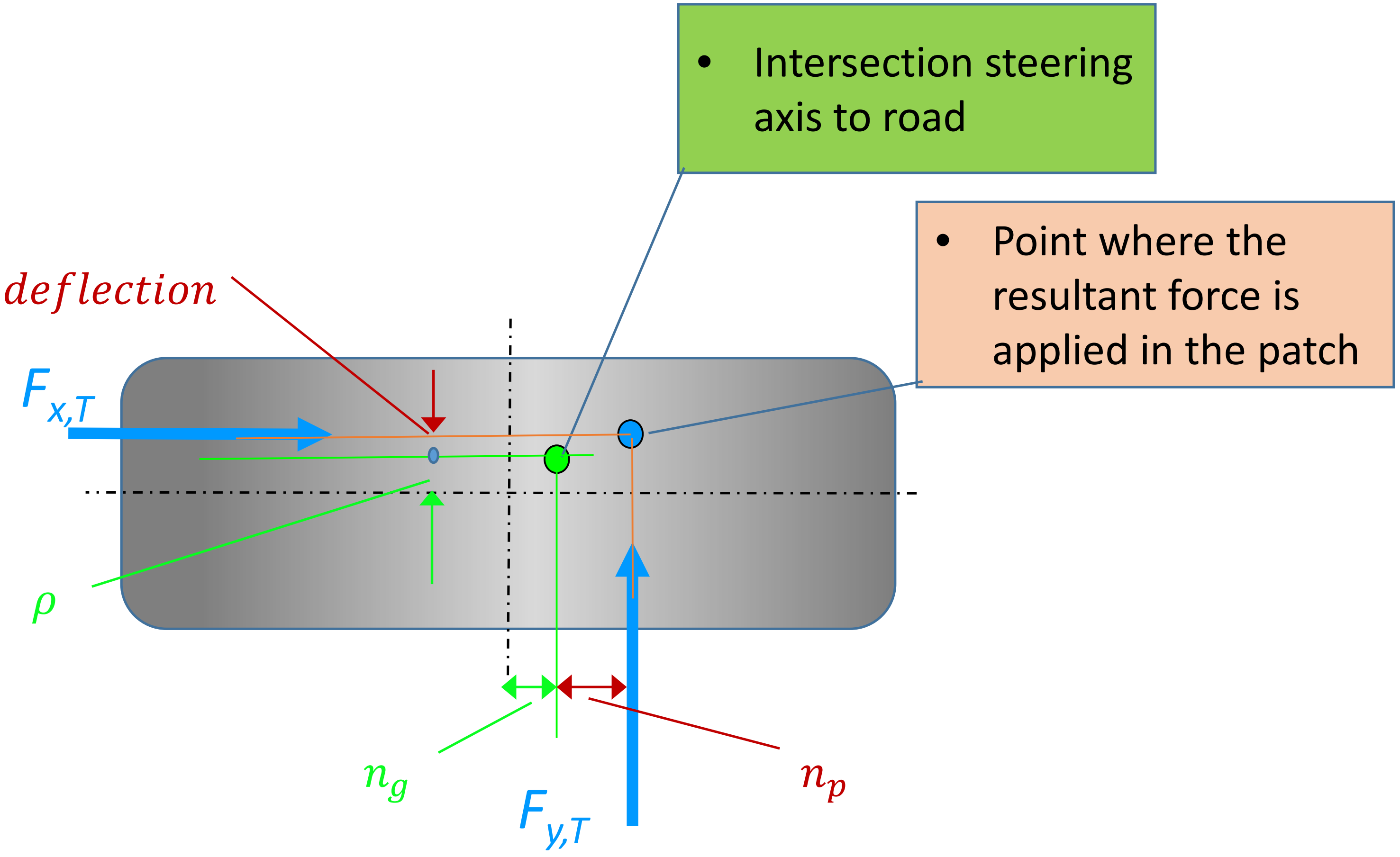
From tire longitudinal force

- king pin inclination angle
- king pin offset

→Scrub Radius

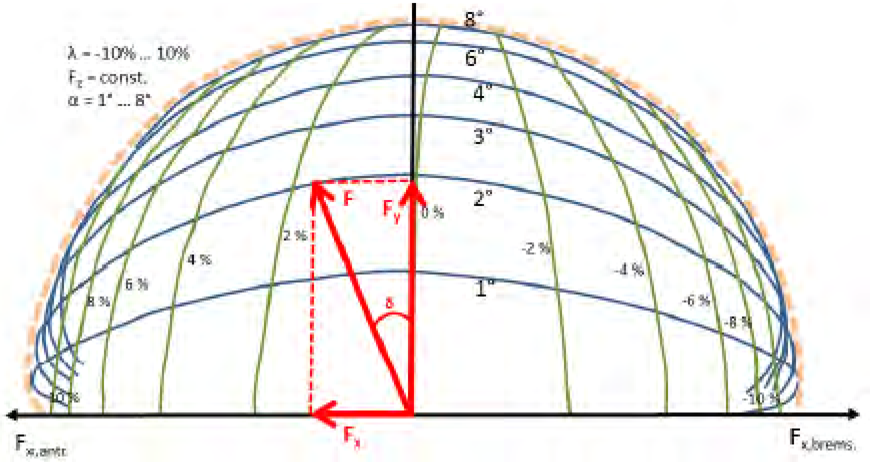
- tire deformation

Wheel Steering Moment due to Combined Forces



„Kamm‘s Friction Circle“

$$\sqrt{\left(\frac{F_x / F_z}{\mu_{x,Max}}\right)^2 + \left(\frac{F_y / F_z}{\mu_{y,Max}}\right)^2} \leq 1$$

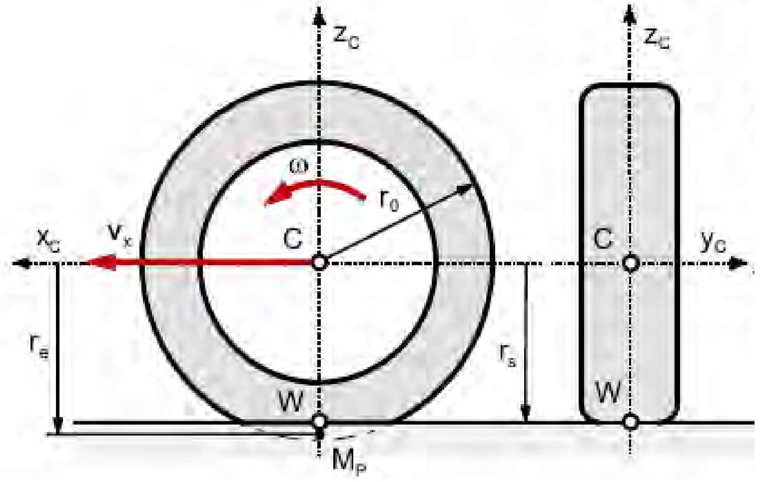


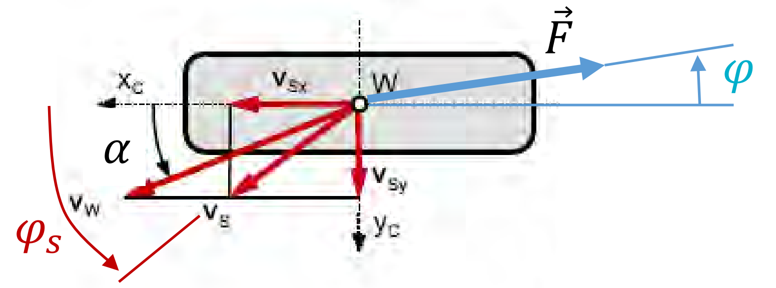
- Wunibald Kamm, 1893 - 1966
  - The geometric sum of longitudinal and lateral force must be within a circle.
- (Krempel‘s improvement)
  - $F_{y,\mathrm{max}} < F_{x,max}$
  - Usually this is also called Kamm‘s friction circle.
  - $\sqrt{\left(\frac{\mu_x}{\mu_{x,max}}\right)^2 + \left(\frac{\mu_y}{\mu_{y,max}}\right)^2} \leq 1, \mu_i = \frac{F_i}{F_z}$

Wheel Slip – Slightly different Definitions

- Mitschke-Wallentowitz (2003)
  relative speed over impressed speed
  - $s_x = \frac{r_{eff} \cdot \omega - v_x}{\max(r_{eff} \cdot \omega, v_x)} = \frac{v_{Sx}}{\max(r_{eff} \cdot \omega, v_x)}$,
  - $-1 \leq s_x \leq 1$
- Mitschke, Pacejka $s_y = \frac{v_y}{vx} = \tan(\alpha)$

- 3 important angles
  - Tire slip angle α
  - Force angle φ
  - angle of relative velocity print to road φ_s
  - Due to anisotropic tire: $\varphi_s > \varphi$

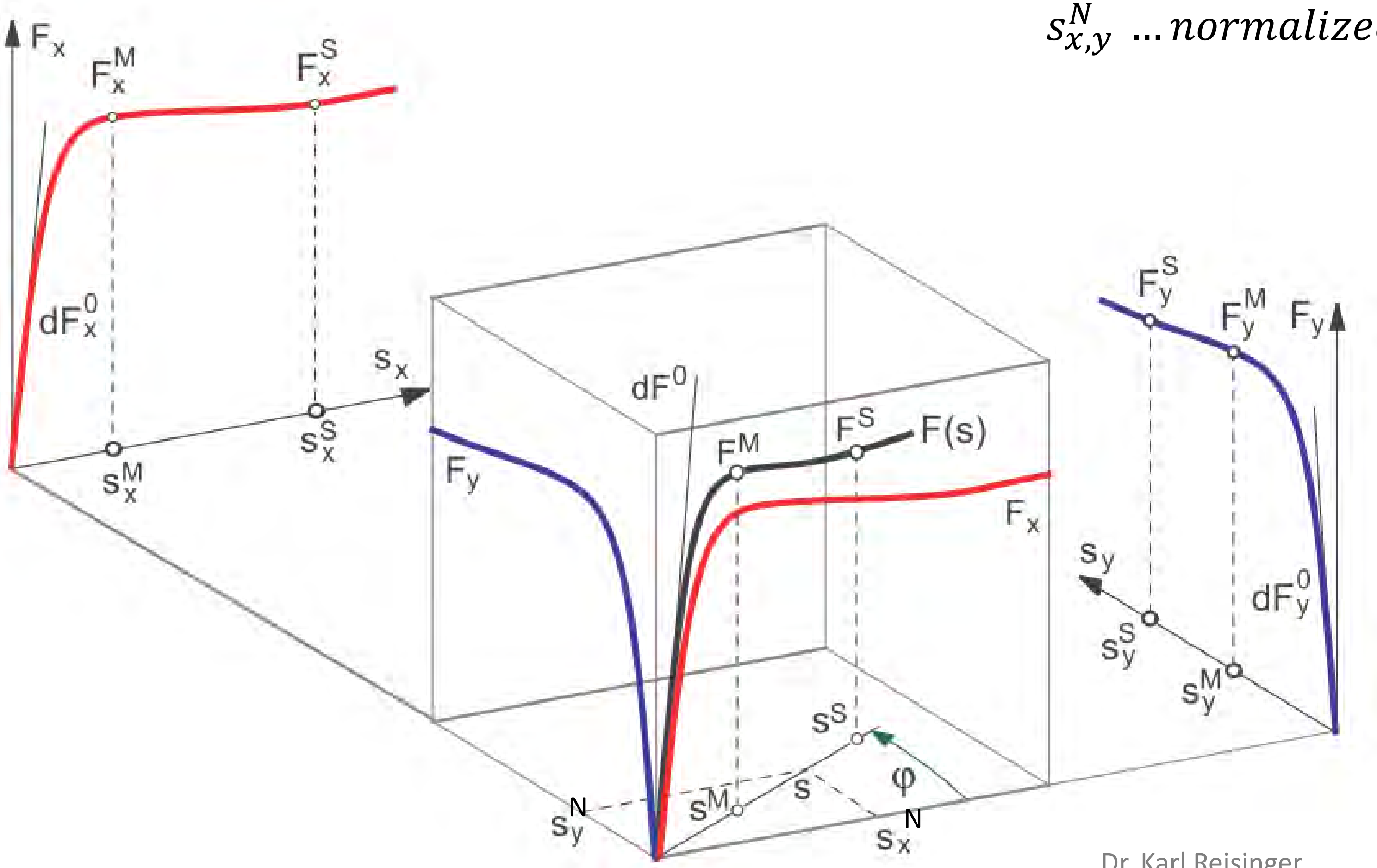




Velocities at the wheel [Hirschberg06]

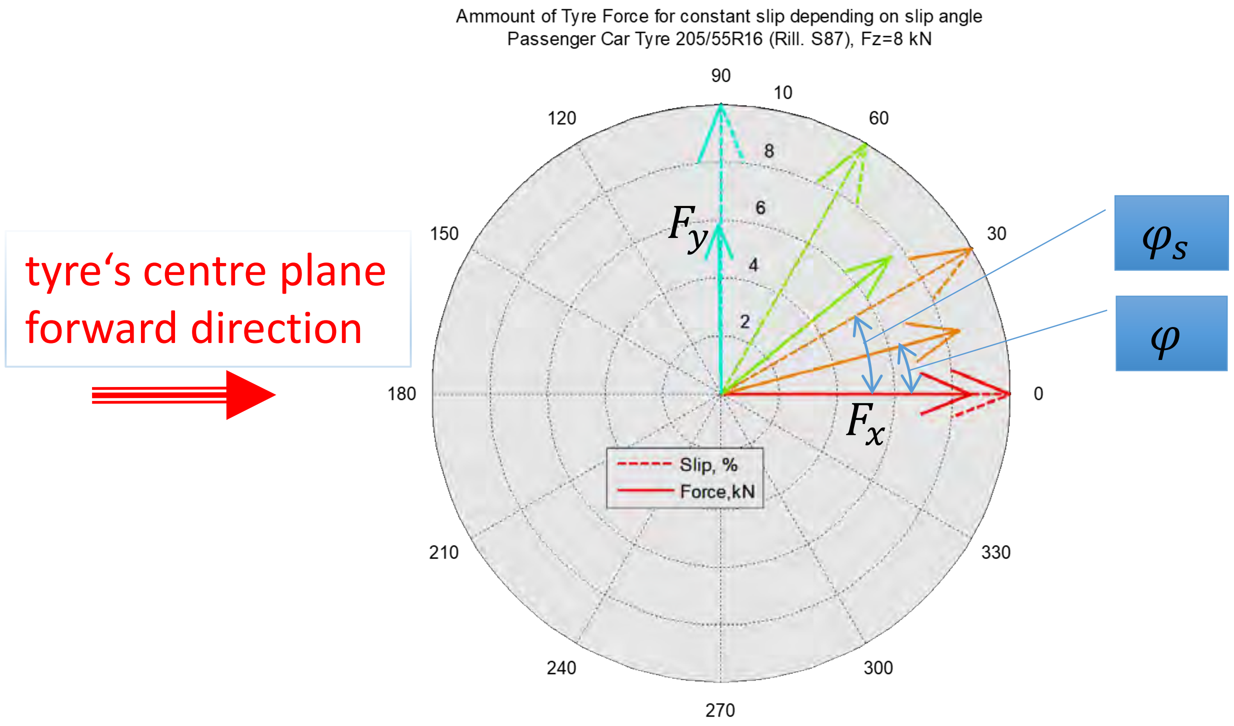
Combined Slip and Forces

$s_{x,y}^{N}$ ...*normalized Slip*



[Rill G.]

Different Directions of slip and force at 10% slip



$$
\begin{array}{ll}
F_z = 8.0\,kN & F_z = 8.0\,kN \\
dF_x^0 = 200\,kN & dF_y^0 = 80\,kN \\
s_x^M = 0.100 & s_y^M = 0.220 \\
F_x^M = 8.70\,kN & F_y^M = 7.50\,kN \\
s_x^S = 0.800 & s_y^S = 1.000 \\
F_x^S = 7.60\,kN & F_y^S = 7.40\,kN
\end{array}
$$

$$\varphi \neq \alpha, \varphi \neq \varphi_s$$

α... Angle between wheel centre‘s velocity and wheel‘s centre plane

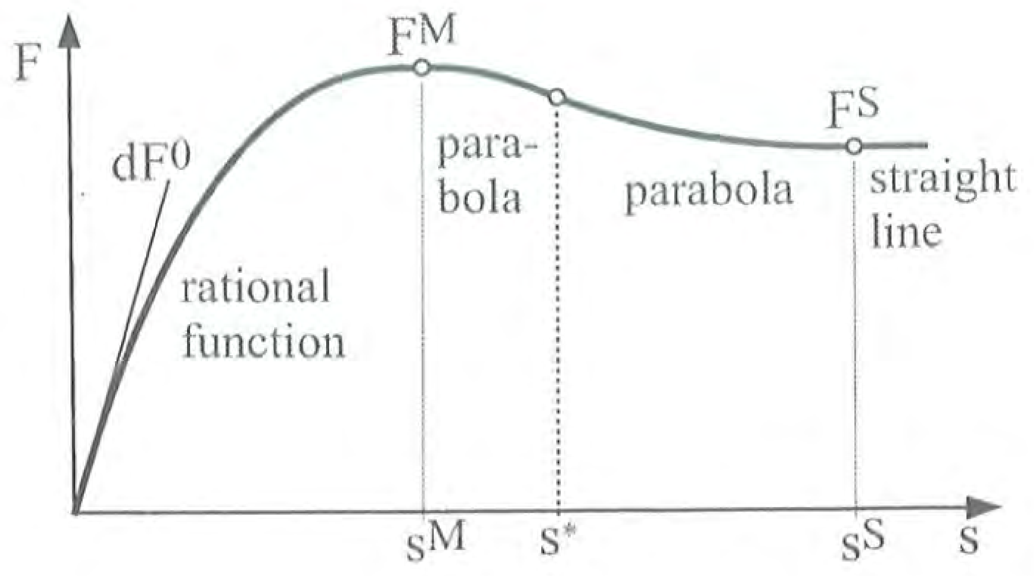
φ_s...Angle between velocity of footprint and wheel‘s centre plane

φ... Angle between contact force and wheel’s centre plane

Semi-Empirical Tire model
Hirschberg-Rill TM-Easy: F(s)

- Parameters
- $dF^0 = \left.\frac{dF}{ds}\right|_{s=0}$ .. Stiffness
- (s^M, F^M).. Maximum
- (s^S, F^S).. Begin of Slide
- Equation

- $F(s) = \begin{cases} \dfrac{s}{1+\frac{s}{sM}\left(\frac{s}{sM}+\frac{dF^0 s^M}{F^M}-2\right)} dF^0 \\ F^M - a(s-s^M)^2 \\ F^S + b(s^S - s)^2 \\ F^S \end{cases}$



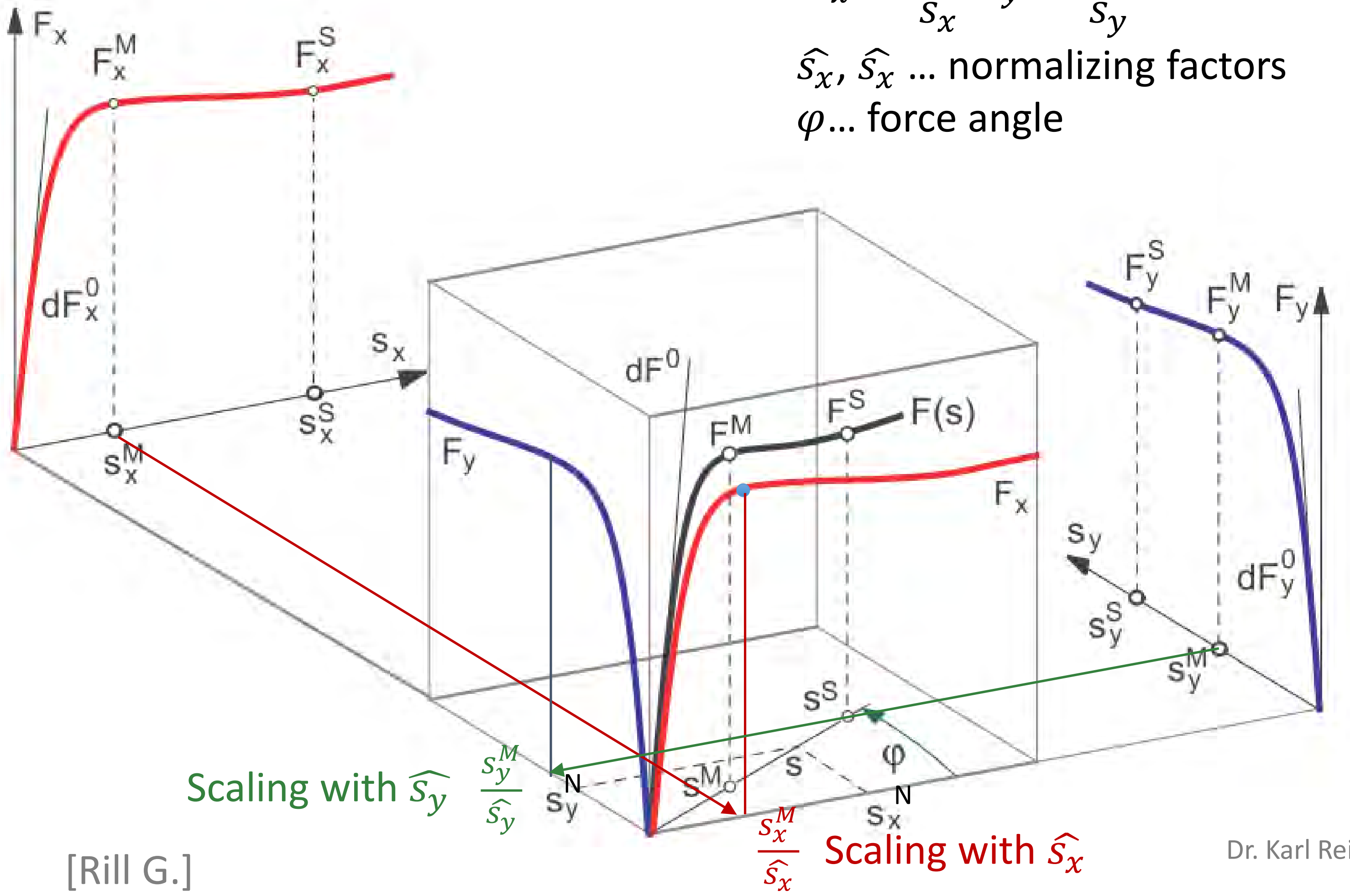
[Rill G.]

Combined Slip and Forces

$s_x^N = \frac{s_x}{\widehat{s_x}}, s_y^N = \frac{s_y}{\widehat{s_y}} \ldots normalized\ slip$

$\widehat{s_x}$, $\widehat{s_x}$ … normalizing factors

φ… force angle



[Rill G.]

Hirschberg-Rill TM-Easy: Combined Forces 1

- Normalized Slip
- $s_x^N = \frac{s_x}{\widehat{s_x}}, s_y^N = \frac{s_y}{\widehat{s_y}}$
- Slip Normalizing Factors $\widehat{s_{x,y}} = f(s_x^M, s_y^M, F_x^M, F_y^M, dF_x^0, dF_y^0)$
- considers, that the tyre is weaker in y than in x
- Resultant Slip
- $s = \sqrt{(s_x^N)^2 + \left(s_y^N\right)^2}$

- But: Normalization is not necessary, if $F_x = s_x = 0$ or $F_y = s_y = 0$

[Rill G.]

Normalized Slip is watched in $\varphi - Plane$

= Force Plane

$$F_x = F\cos\varphi \quad \text{and} \quad F_y = F\sin\varphi$$

$$F(s) = \begin{cases} s^M dF^0 \dfrac{\sigma}{1+\sigma\left(\sigma + dF^0 \dfrac{s^M}{F^M} - 2\right)}, & \sigma = \dfrac{s}{s^M}, \quad 0 \le s \le s^M \\ F^M - (F^M - F^G)\sigma^2(3-2\sigma), & \sigma = \dfrac{s - s^M}{s^G - s^M}, \quad s^M < s \le s^G \\ F^G, & s > s^G \end{cases}$$

$$dF^0 = \sqrt{\left(dF_x^0 \hat{s_x} \cos\varphi\right)^2 + \left(dF_y^0 \hat{s_y} \sin\varphi\right)^2},$$

$$s^M = \sqrt{\left(\frac{s_x^M}{\hat{s_x}}\cos\varphi\right)^2 + \left(\frac{s_y^M}{\hat{s_y}}\sin\varphi\right)^2},$$

$$F^M = \sqrt{\left(F_x^M \cos\varphi\right)^2 + \left(F_y^M \sin\varphi\right)^2},$$

$$s^G = \sqrt{\left(\frac{s_x^G}{\hat{s_x}}\cos\varphi\right)^2 + \left(\frac{s_y^G}{\hat{s_y}}\sin\varphi\right)^2},$$

$$F^G = \sqrt{\left(F_x^G \cos\varphi\right)^2 + \left(F_y^G \sin\varphi\right)^2}$$

$$\cos\varphi = \frac{s_x/\hat{s_x}}{s} \quad \text{and} \quad \sin\varphi = \frac{s_y/\hat{s_y}}{s}$$

[Rill G.]

Hirschberg-Rill TM-Easy: Load Dependence

- Force parameters dF^0, F^M, F^S: quadratic rule
- $Y(F_z) = \frac{F_z}{F_z^N}\left\{2\,Y(F_z^N) - \frac{1}{2}\,Y(2F_z^N) - \left[Y(F_z^N) - \frac{1}{2}Y(2F_z^N)\right]\frac{F_z}{F_z^N}\right\}$
- $\rightarrow \mu(F_z) = \frac{Y(F_z)}{F_z}$... linear interpolation

- Slip parameters s^M, s^S: linear rule
- $X(F_z) = X(F_z^N) + \left[X(2F_z^N) - X(F_z^N)\right]\left(\frac{F_z}{F_z^N} - 1\right)$

[Rill G.]

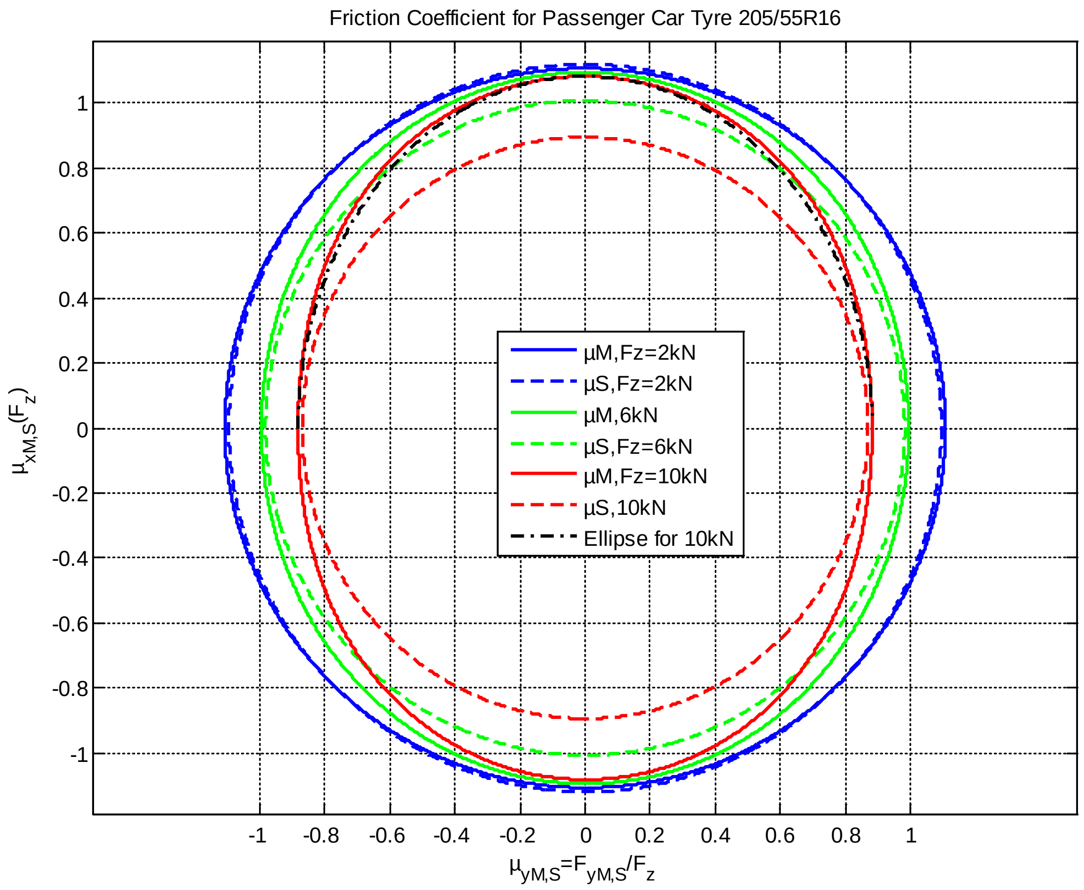
Homework for students

- Please read the paper provided on moodle:
- Hirschberg_Rill_Weinfurter_Tire model TM-Simple_User-Apporpriate Tyre-Modelling for Vehicle Dynamics in Standard and Limit Situ.pdf

Hirschberg W., Rill G.: User-Appropriate Tyre-Modelling for Vehicle Dynamics in Standard and Limit Situations, Vehicle System Dynamics, Vol. 38, 2002, Issue 2, Pages 103-125 | Published online: 09 Aug 2010

Friction Circe depending on $\mu_{m,s}(F_z)$

- $\mu M = \frac{F^M}{F_z}$ .. max. friction, μS.. sliding friction coefficient passenger car tyre, result of TM-Easy



Semi-Empirical Tyre model Pacejka's famous Magic Formula [1)]

also for large road wave-lengths only!

- " a distorted" Sine function
- $Y(x) = D sin[\arctan(\mathrm{B}\Phi)] + \mathrm{S_v}$
- $\Phi = (1 - E)\, x + \left(\frac{E}{B}\right) \arctan(B\, x)$
- $x = \kappa + S_h, x = \alpha + S_h$
- $\kappa_x = \frac{r_e \omega - v_x}{v_x}$ … slip ratio, α… sideslip angle
- v_x… velocity of wheel centre in direction of centre plane =x-direction
- B, C, D, E, S_h, S_v… parameters to fit the behaviour, those are different polynomic functions of F_z, inclination angle ("camber") and air pressure.
- The Magic Formula can describe $\mu_x(\kappa), \mu_y(\alpha), M_z(\alpha)$.
- 1) introduction: Bakker E.,Pacejka H.,Lindner L.: A New Tire Model with an Application in Vehicle Dynamics, SAE April 1989

Pacejka Magic Formula: Meaning of Parameters

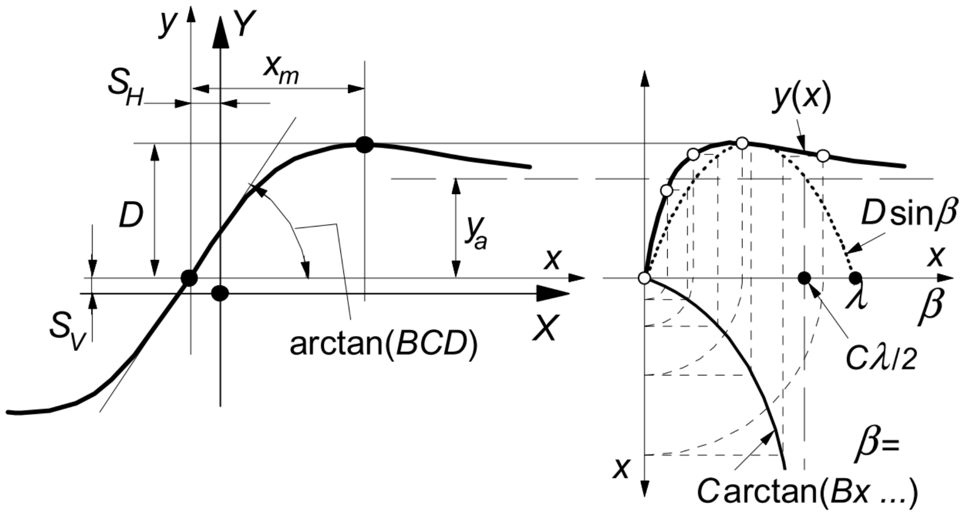


Fig. 4.9. Curve produced by the original sine version of the Magic Formula, Eq.(4.49). The meaning of curve parameters have been indicated.

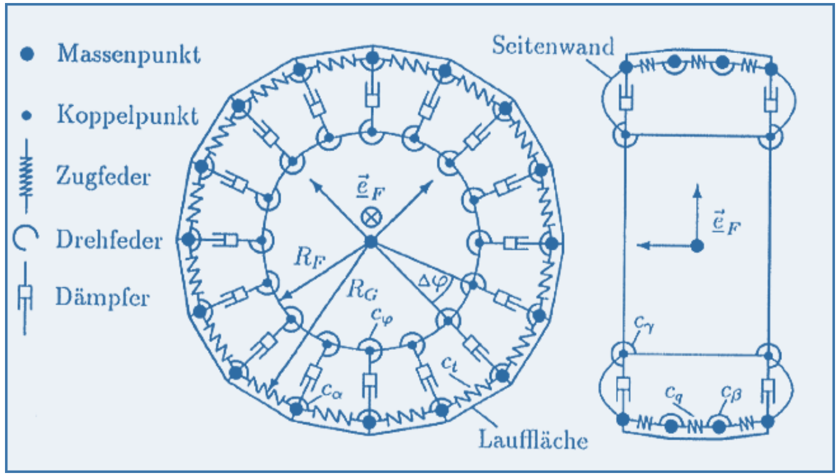
- D … Peak
- BCD … stiffness
- Sv, Sh … asymmetry
  - eg: Sv due to camber
  - Sh due to asymmetric profile

Pacejka's load dependence

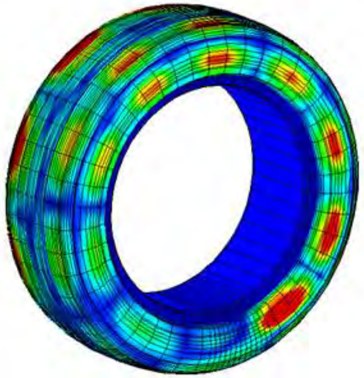
- Different Fit-function for each parameters, e.g.
  - $D = a_1 F_z^2 + a_2 F_z$
  - $BCD = \frac{a_3 F_z^2 + a_4 F_z}{e^{a_5 F_z}}$
- Process to get Tyre Model
  - Fit B, C, D, E, S_h, S_v using Magic Formula for each F_z
  - Fit parameters $a_1, a_2, \ldots$ using special load functions.
  - Different Fit-Functions depending on version of Pacejka Model

Tire models: as fine as needed …

- Characteristic models, (real time capable)
  - ” Magic Formula” (Hans Pacejka),
  - TM-Simple, TM-Easy (Hirschberg-Rill)
  - Unevenness with wide wave-lengths
  - 1/Curvature of road > r_e
- Lumped Mass Models, MKS Simulation
  - MKS models having rigid elements connected with springs and dampers,
  - e.g. RMOD-K, F-Tyre, …
  - Offroad, curb stone edge
- Continua, FEM Models
  - NVH Simulation
  - Tyre Development

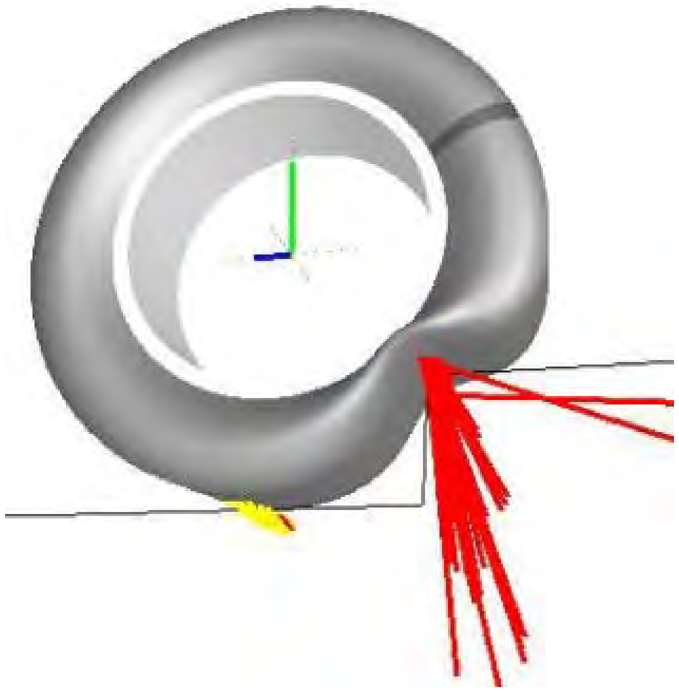


MKS tyre model [Mitschke04]



https://www.tuhh.de/forschung/fobe/2005/a1998.5-03/w.67.1129626881235.html

F-Tire at curb stone edge



[Gipser: Reifenmodelle i.d. Fahrdynamik, wikipedia]

Engineering Knowledge Transfer Units to Increase Student´s Employability and Regional Development

Longitudinal dynamics

Single track model, transient and steady state tests, Backward Sim. Models, Forward Sim. Models.

Co-funded by the Erasmus+ Programme of the European Union

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Duties of long. Dynamics

- Drag forces (aerodynamic, rolling)
- How fast accelerates a car due to an engine torque?
- How fast can a car accelerate due to tire?
- Energy consumption to drive a certain cycle?
- The Traction-Force Effort Diagram to show car’s capability

Drag Force (Fahrwiderstand)

Static drag forces at horizontal road	Our assumption	Comment
Climbing resistance	$F_C \cong m_{veh} \cdot g \cdot \sin(\alpha)$	very high
Aerodynamic Drag	$F_{AD} = c_{wx} \cdot A_x \cdot \frac{\rho}{2} v^2$	Largest in horizontal road above 40-70 kph
Rolling Force, Rolling Drag	$F_R \cong f_R \cdot m_{veh} \cdot g \cdot \cos(\alpha)$	Largest at low speed, very high in mud & sand; In racing, aerodyn. downforce must be considered.
Mechanical drag losses due to brakes and wheel bearings	≈ 0	Neglectable with proper working brakes and low preload at bearings.
Losses in drivetrain due to bearings, …	≈ 0	e.g. a preloaded taper roller bearing of input shaft of read axle differential costs 0.9% of traction energy.
Damper induced drag	≈ 0	Very low at regular roads, plays a rule Offroad; Compare Putzik 2008
toe induced resistance	≈ 0	must be adjusted within some angle minutes
Curve induced resistance (lateral velocity at tire times side force consumes power)	≈ 0	Low with low lateral accelerations. Usually not considered in consumption models.

α … Road inclination angle

Aerodynamic Drag

- Force = Velocity Pressure x Air Drag Coefficient x Projected Area
- $F_{wx} = \frac{\rho_{Air}}{2} \left(v_{veh,x} + v_{amb,x}\right)^2 \mathrm{c_{wx}}\, \mathrm{A_x}$
- Modern passenger car
  - A_x~2 m^2, c_{wx} ~ 0.3-0.4,
- Commercial vehicle
  - A_x~8 m^2, c_{wx} ~ (0.45)- 0.85
- Motorbike, driver sits up straight
  - A_x~1.0 m^2
  - A_x c_{wx} ~ 0.5 - 0.6 m^2
  - Loremo (Study): c_{wx} =0.2

Loremo [http://www.hybridantrieb.org]

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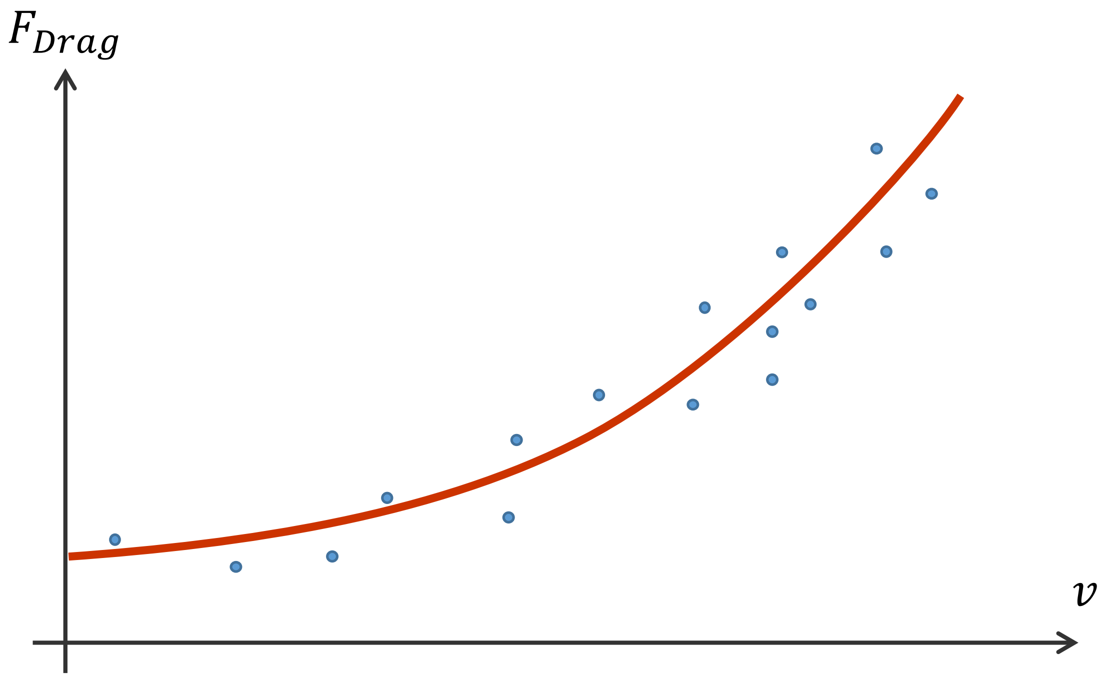
Drag Measurement

- **Coast Down Test at horizontal road in Neutral Gear measures**
  - Rolling resistance
  - + Aerodynamic Drag
  - + losses in drive train
- Measure speed over time
- Differentiate in respect to time, calc. drag

$$(m_{veh} + m'_{rot}) \cdot \frac{d\,v}{dt} = F_{Drag} \cong F_R + F_{AD}$$

- Fit quadratic parabolic equation

$$F_{Drag} = A + B \cdot v + C \cdot v^2$$



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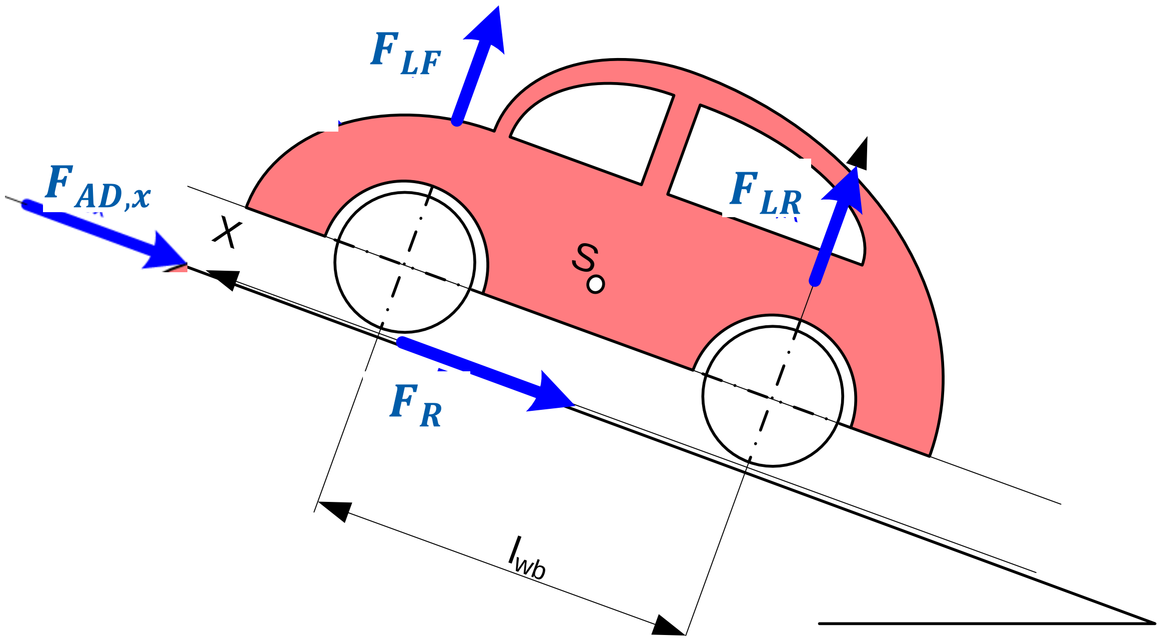
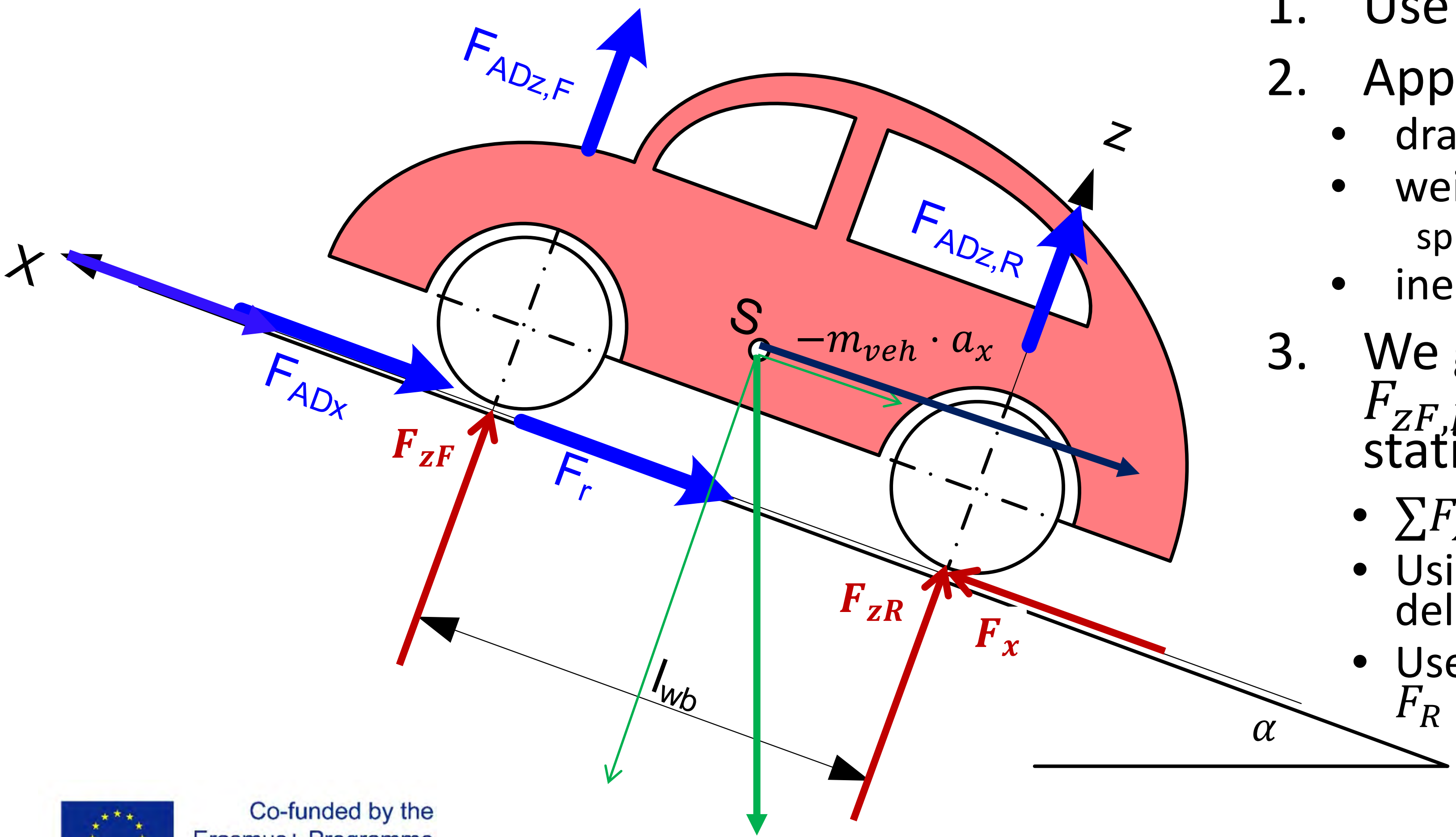
Aerodynamic Lift

- Relevant at racing cars
- Wind tunnel or simulation
  - Measure forces at tire prints
  - $F_{\mathrm{AD}x}, F_{\mathrm{AD}z,F}, F_{\mathrm{AD}z,R}$
  - Divide by A_x oder A_z $\rightarrow c_{wx}, c_{\mathrm{AD}zF}, c_{\mathrm{AD}zR}$
    - $F_{\mathrm{AD}F/R} = \frac{\rho_{Air}}{2}\left(v_{veh,x} + v_{amb,x}\right)^2 \mathrm{c}_{\mathrm{AD}F/R}\,\mathrm{A_x}$ or
    - $F_{\mathrm{AD}zF/R} = \frac{\rho_{Air}}{2}\left(v_{veh,x} + v_{amb,x}\right)^2 \mathrm{c}_{\mathrm{AD}F/R}\,\mathrm{A_z}$
  - Attention: Different literature uses different nominal area $A_x$or A_z
  - If lift is considered, drag applies at road level.
- No lift data?
  - estimate c_{wx}
  - Estimate centre of aero drag force application $F_{\mathrm{AD}x}$ for $M_{\mathrm{AD}y} = F_{AD\mathrm{x}} \cdot h_{AD}$



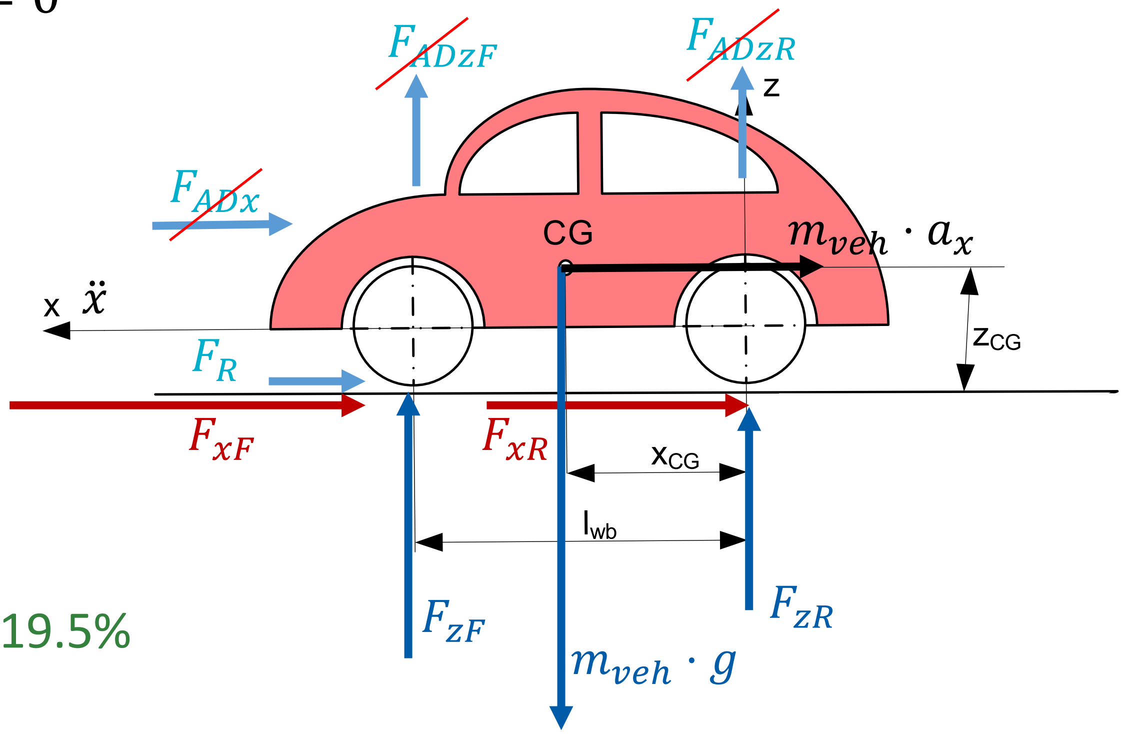
d‘Allembert‘s view delivers simple equations



1. Use CS in road plane
2. Applied loads
   - drag and lift forces
   - weight $G = m_{veh} \cdot g$
     split into components
   - inertia force $-m_{veh} \cdot a_x$
3. We get dynamic wheel loads $F_{zF,R}$ and traction force F_x by semi-static equations
   - $\sum F_X = 0, \sum F_z = 0, \sum M_Y = 0$
   - Using wheel contact for sum of moments delivers a low number of terms.
   - Use $F_R = (m_{veh} \cdot g \cdot \cos(\alpha) + F_{LF} + F_{LR})$

Example
Wheel Load Distribution, flat road

- $+m\ g\ x_{CG} - m\ a_x\ z_{CG} - F_{zF}\ l_{wb} = 0$ with $F_{wzF} = 0$
- $F_{zF} = m\ g \frac{x_{CG}}{l_{wb}} - m\ a_x \frac{z_{CG}}{l_{wb}}$ … Lin. Equation
- $\frac{F_{zF}}{m\ g} = \frac{x_{CG}}{l_{wb}} - \frac{a}{g} \frac{z_{CG}}{l_{wb}}$
- $\frac{F_{zR}}{m\ g} = 1 - \frac{x_{CG}}{l_{wb}} + \frac{a_x}{g} \frac{z_{CG}}{l_{wb}} = 1 - \frac{F_{zF}}{m\ g}$
- Static Distribution Front: $\frac{F_{zF0}}{m\ g} = \frac{x_{CG}}{l_{wb}} = 0.56$
- Static Distribution Front/Rear : 56%/44%
- Dynamic Distribution Front/Rear @$-10\,\frac{\mathrm{m}}{\mathrm{s}^2}$:80.5%/19.5%
- Optimal Brake Distribution: In ratio to wheel load (neglecting tire‘s nonlinearity)



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Influence of rot. Inertia

- When accelerating, we put kinetic energy into rotating parts.
- How much mass we have to put into the boot to have the same behavior as the rot. Inertias?

The rotating mass is to be accelerated by the engine but not by the tire!

$$m_{veh} \cdot a_x = -F_{Drag} - F_{climb} - F_{xTire}$$

$$(m_{veh} + m_{rot}) \cdot a_x = -F_{Drag} - F_{climb} - F_{xDrive}$$

- $E_{kin,rot} = \sum \frac{1}{2} J_i \, \omega_i^2 = \mathrm{E_{kin,mrot}}$
- J .. Inertia of flywheel/rotor, wheels
- $\omega_{wheel} \cong \frac{v_x}{r_e}$
- $\omega_{eng} = \omega_{wheel} \cdot i_{gear}$
- $\frac{1}{2} m_{rot} v_x^2 = \frac{1}{2} \cdot [J_{eng} \cdot \omega_{eng}^2 + 4 \cdot J_{wheel} \cdot \omega_{wheel}^2]$

$$m_{rot} = \left(J_{eng} \cdot i_{gear}^2 + 4 \cdot J_{wheel}\right) \cdot \frac{1}{r_e^2}$$

  - The more important, the faster it runs
  - More important in lower gears
- $m_{tot} = m_{veh} + m_{rot} = \lambda \cdot m_{veh},$ $1.0 < \lambda < (1.4)$

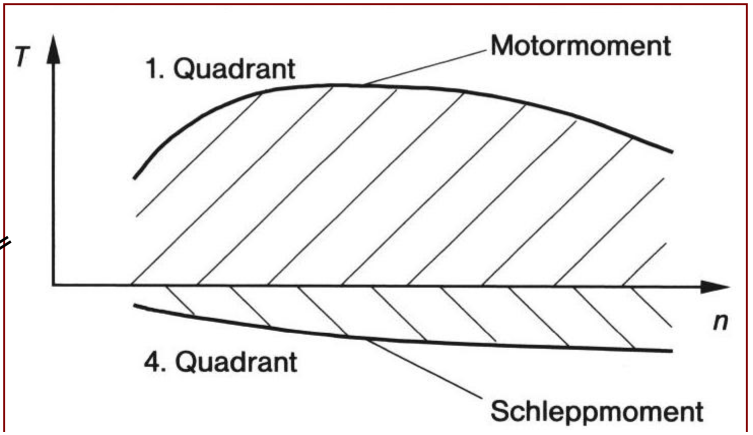
Influence of gear ratio and efficiency

- An ideal gearbox is a transducer where input and output power is equal.
- $P = M \cdot \omega$
  - High speed, low torque
  - Low speed, high torque
- Usually engine size is defined by torque → fast small engine.
  - $\omega_{eng} > \omega_{wheel}, \omega_{eng} = i_{gear} \cdot \omega_{wheel}$
  - $i_{gear} > 1$

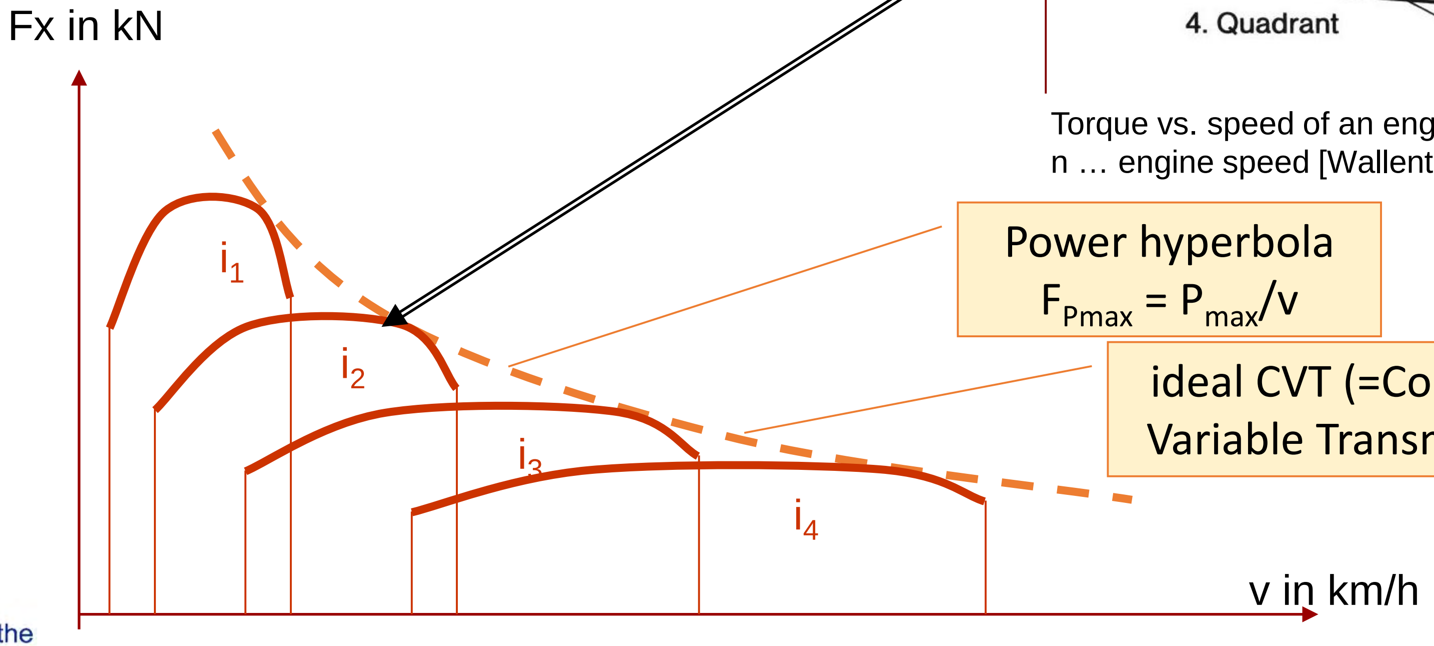
- Efficiency $\eta_{gear} = \frac{P_{out}}{P_{in}}$
  - Useful output over needed input.
  - Non load dependent losses often neglected.
- Accelerating, Thrust Mode
  - The engine delivers power to the wheel; In: P_{eng}, Out: P_{wheel}
  - $P_{wheel} = \eta_{gear} \cdot P_{eng}$
- Braking with engine, coast mode
  - The wheel delivers power to the engine; Out: P_{eng}, In: P_{wheel}
  - $P_{eng} = \eta_{gear} \cdot P_{wheel}$
- Generally
  - $P_{wheel} = \eta_{gear}^{k} \cdot P_{eng}, k = \begin{cases} 1 \,..\, P_{eng} \geq 0 \\ -1 \,..\! < 0 \end{cases}$

Traction Force Diagram

... for non accelerated motion.



Torque vs. speed of an engine, T … engine torque, n … engine speed [Wallentowitz06]



Power hyperbola
$F_{Pmax} = P_{max}/v$

ideal CVT (=Continous Variable Transmission)

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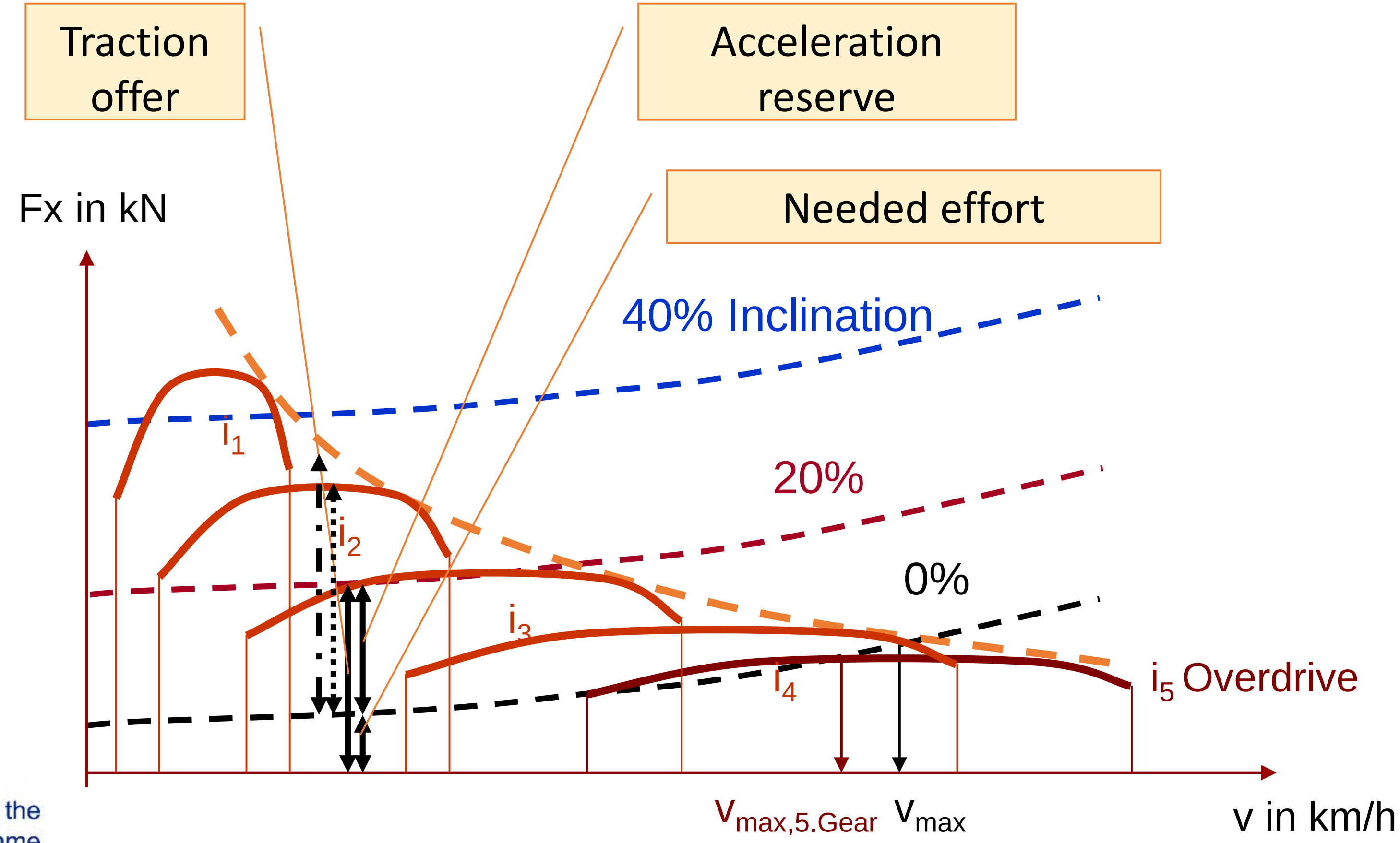
Traction Effort Diagram with Drag

- no acceleration



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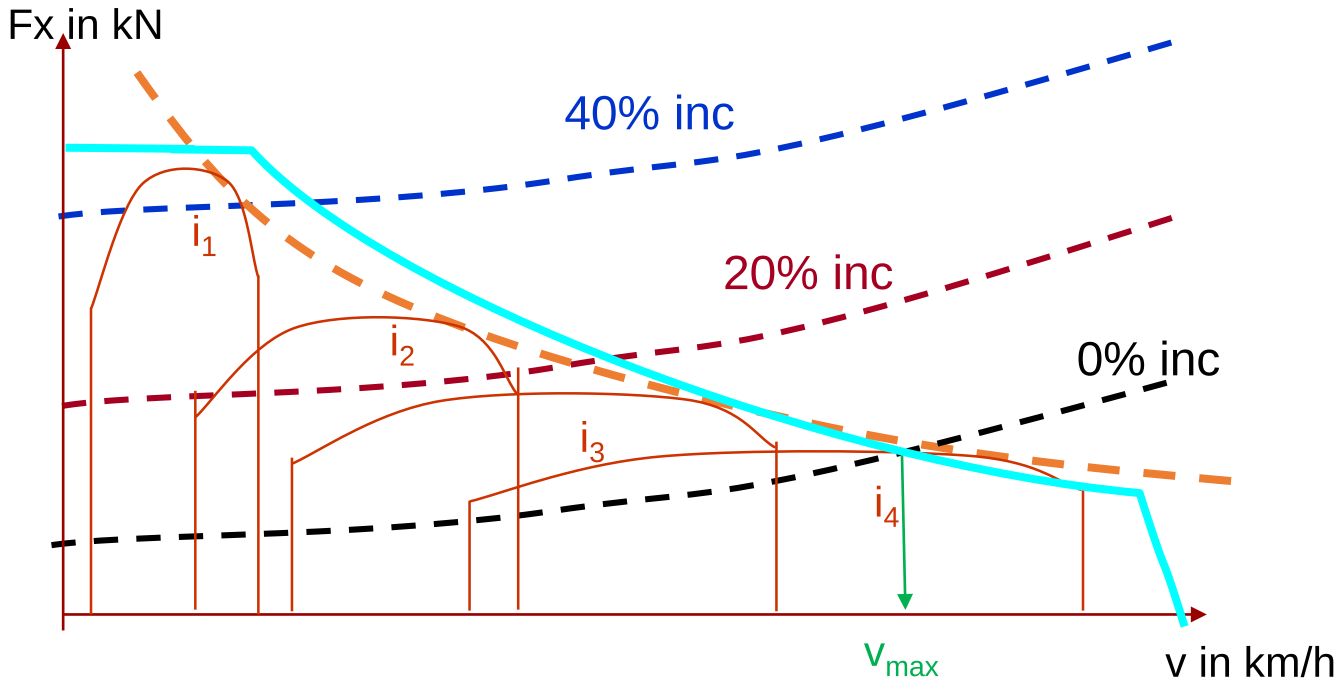
Tractive Effort Diagram, eDrive vs. ICE

- High Starting Torque
  → No Clutch
- M(n) characteristic fits perfect for low to medium speeds
- 2 gears increase efficiency at highway speeds
- Drive can deliver braking torque for recuperation ☺
  → avoid wheel lock up!



cyan: Moment of PMSM with same top speed in 4th gear

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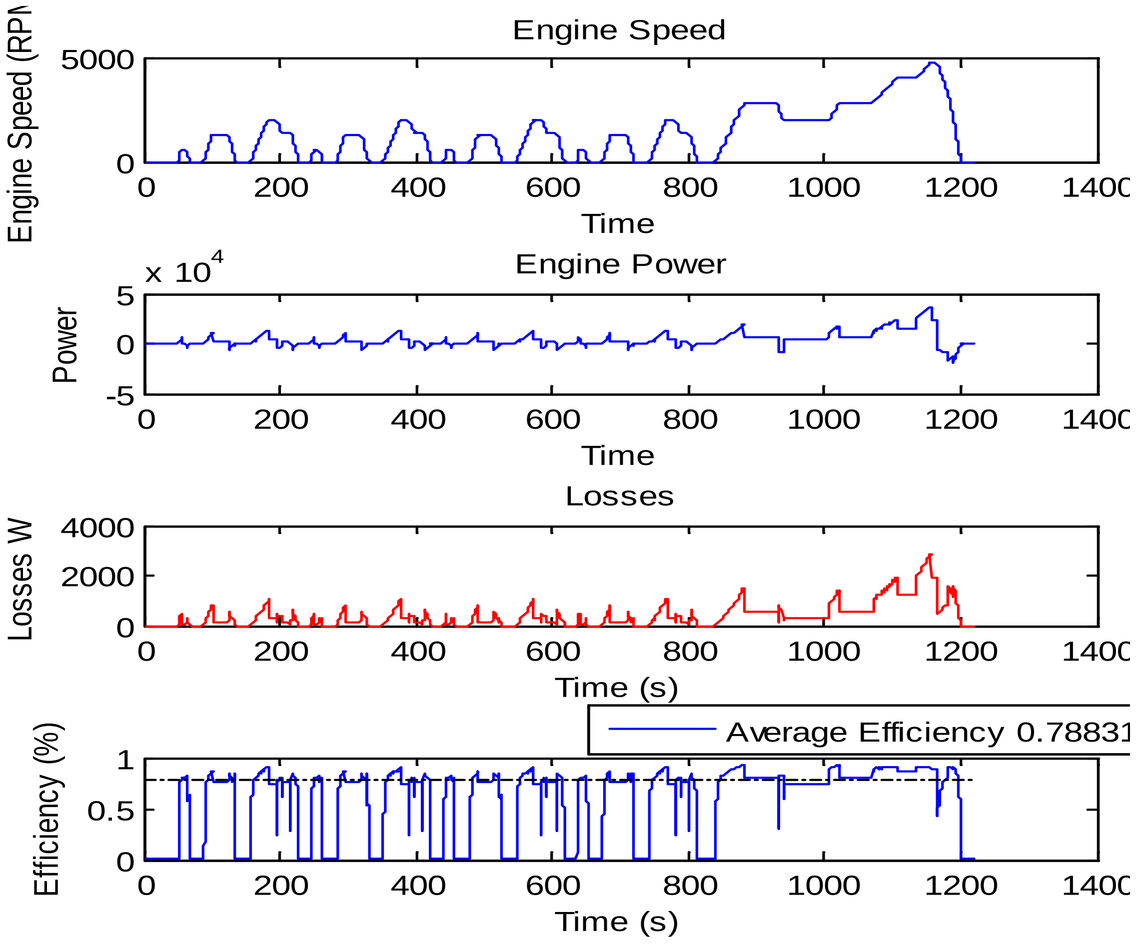
Backward Simulation Model using Requested Trajectory v(t)

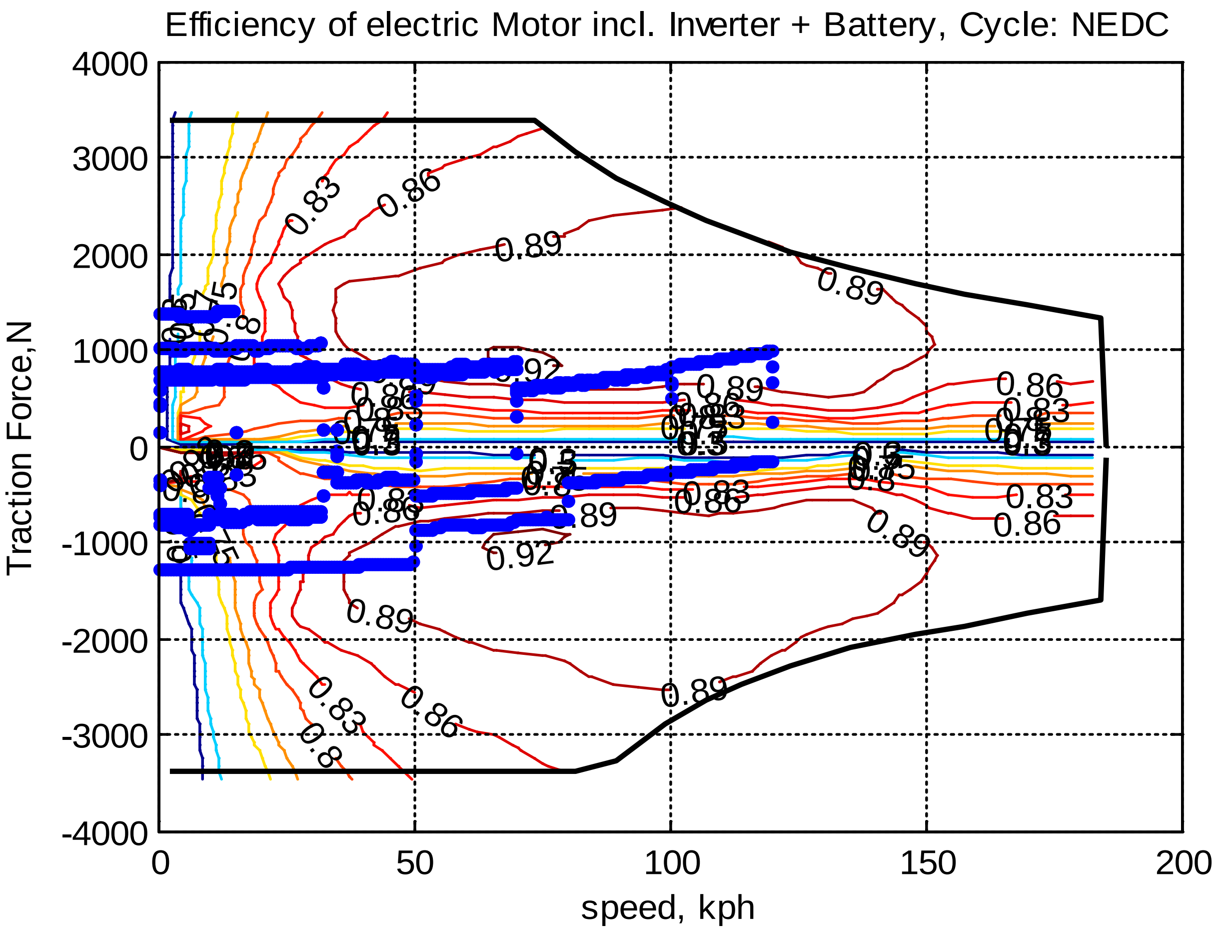
- **Given**
  - **Drivable** Speed Characteristics v(t)
  - road: inclination(s(t))
- **Wanted**
  - Engine torque, speed
  - consumption
  - Forces, torques in drivetrain for fatigue testing
- **Preliminaries**
- Engine power is high enough to follow the requested speed
- Wheels don‘t skid, we are in increasing branch of Fx(sx)

- **Solution**
- Integrate velocity v(t) numerically → distance s(t) used for inclination(s)
- Differentiate velocity v(t) numerically → acceleration a(t)
- Principle of linear momentum →Tyre Traction Force $F_{x,Tyre}$
- Principle of angular momentum → wheel/axle load F_z
- Traction coefficient $\mu_x = \frac{F_{x,Tyre}}{F_z}$
- Inverted tire characteristics → $s_x(\mu_x)$
- Principle of linear momentum → Drive Traction Force $F_{x,Drive}$
- Power at wheel, Power at engine
- Efficiency map/fuel consumption map → fuel/energy consumption

MATLAB (Octave)

NEDC





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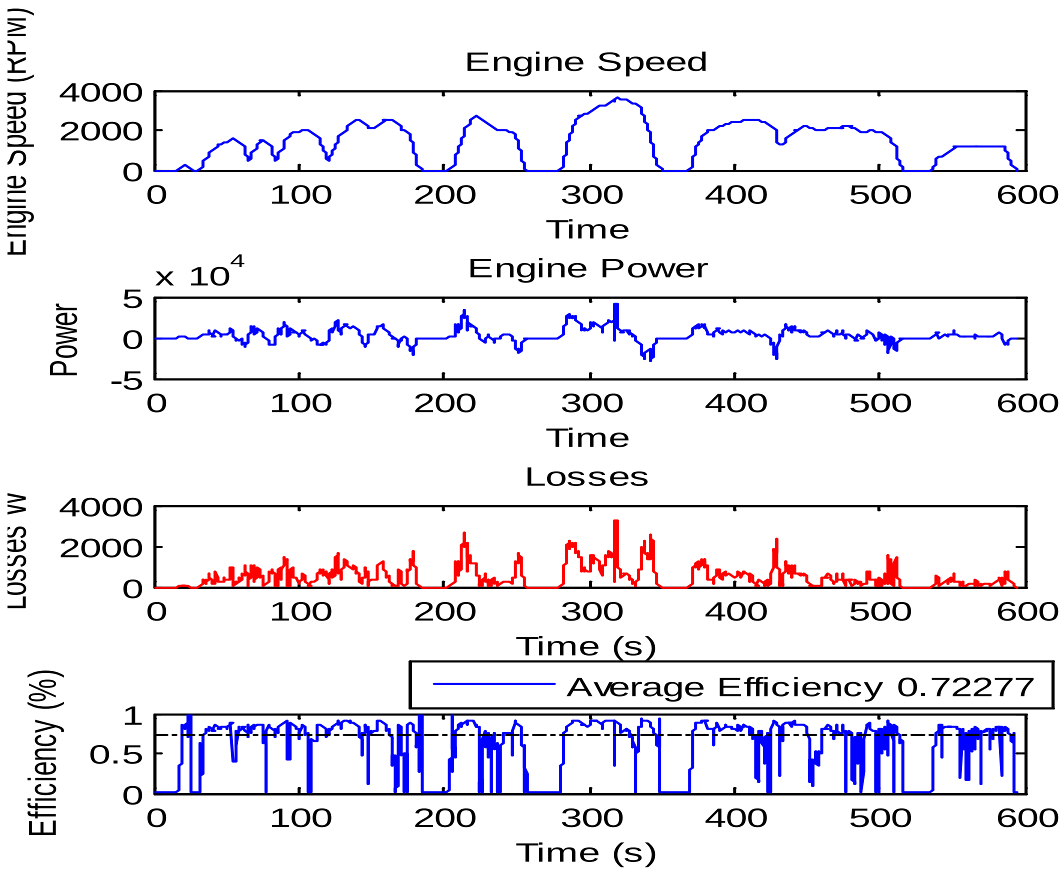

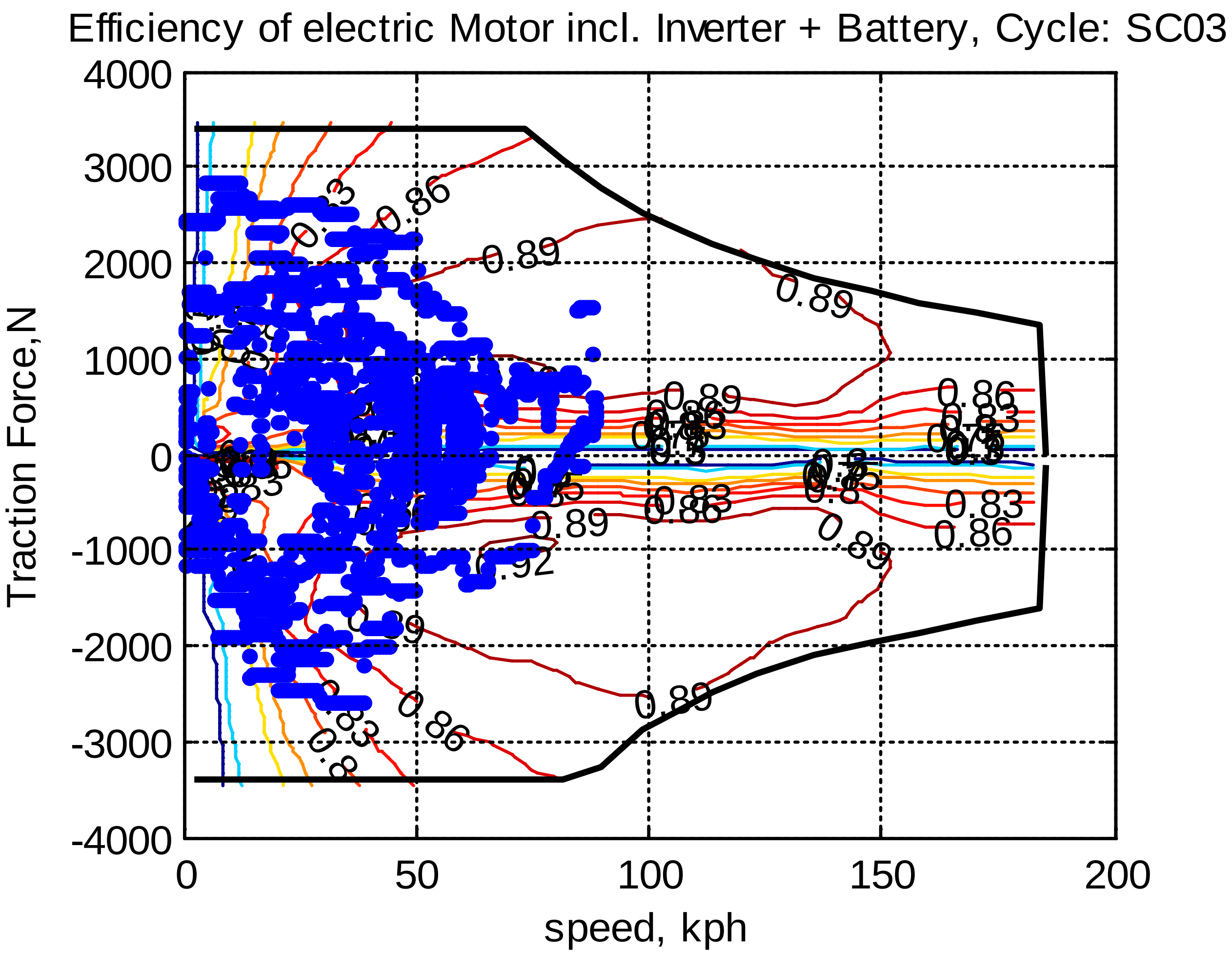

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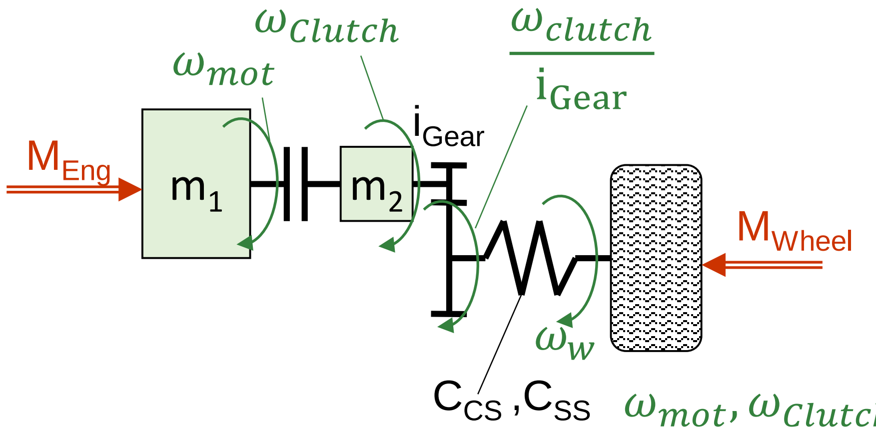
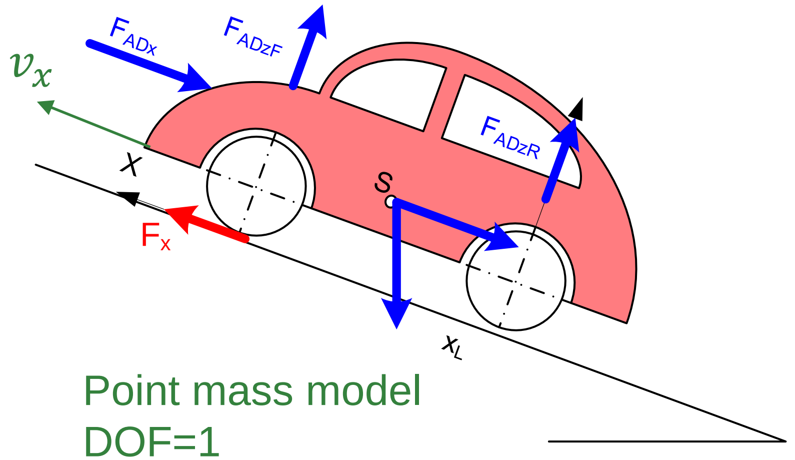
Forward Simulation Model using a Driver

- Driver (PI-Controller)
  - In: $v_{Req}(t), v(t)$, out: Accel. Ped. AP
- Engine
  - Throttle characteristics $M_{Mot0}(nMot, AP)$
  - 1st order delay → M_{Mot}
- Multi Body Simulation Model
  - Rigid bodies, 1 DOF each
    - e.g.: power train: motor, clutch+gear, wheels
    - chassis: 1 DOF in x
  - connected by massless force elements
    - Clutch + torsional springs, side shaft , tire model

Simulink



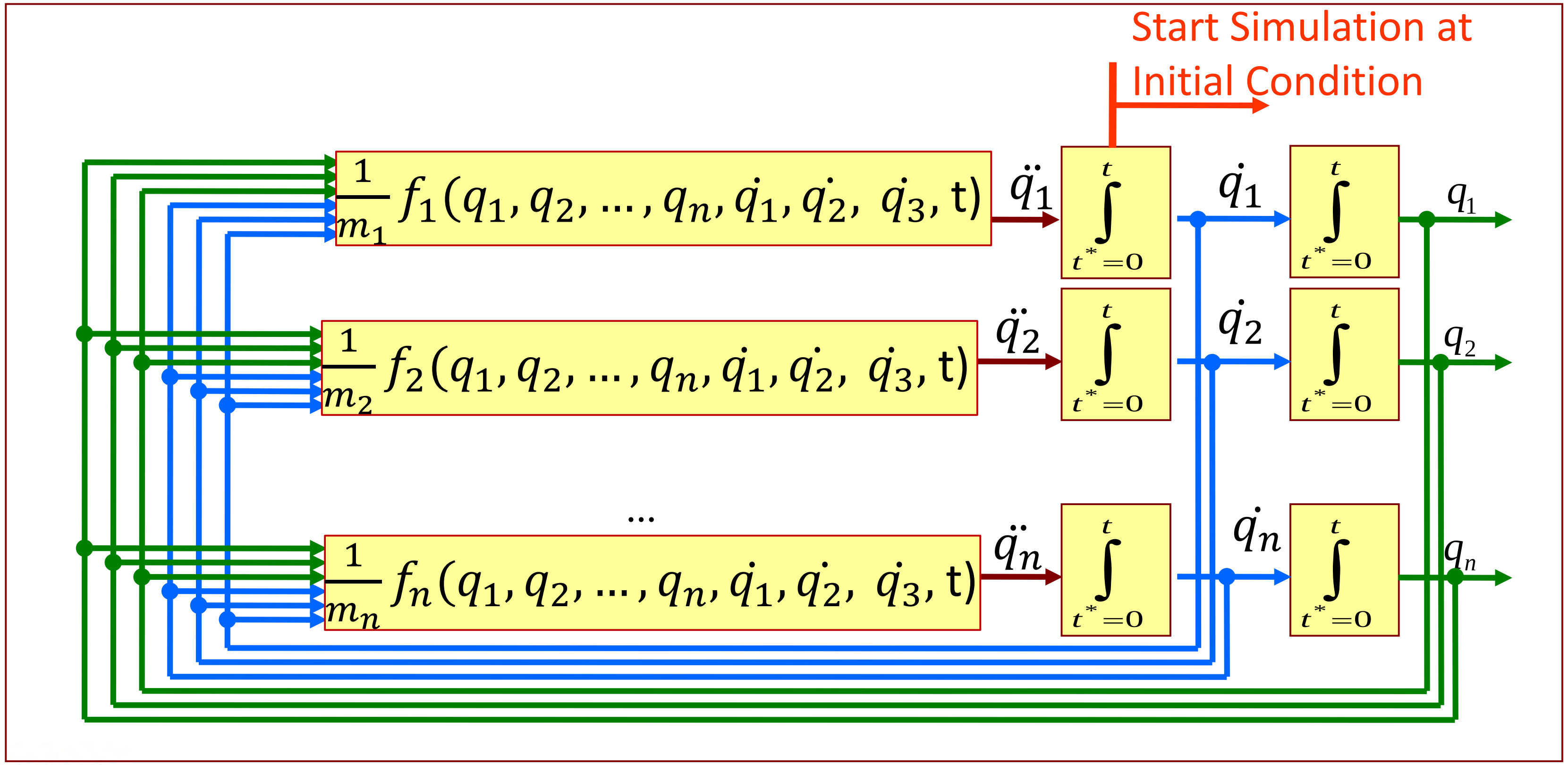
$\omega_{mot}, \omega_{Clutch}, \omega_w, v_x$
.. rot. only, 3 DOF



Point mass model
DOF=1

Principle to solve ODE‘s

- n DOF: n Nonlinear Ordinary Differential Equations of 2nd Order



Practice Backwards Sim. Model

- PracticeBackLong.m

use MATLAB or Octave to run.

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Discussion

- Please form 4 -5 Groups, I propose to mix up, between the universities.

- Discuss following Questions:
  - Other didactic approaches to introduced topics
  - Topics I missed generally (compared to overview sheet)
  - Topics we cancelled, because we don’t think, they are so important.

- Presentation and discussion of your results.

Literature

- Milliken W.F, Milliken D.: Race Car Vehicle Dynamcs,SAE Int., 1995
- Rill G.: Road Vehicle Dynamics – Fundamentals and Modelling; CRC Press Taylor Francis Group
- Heißing/Ersoy (Eds.): Chassis Handbook, 1st Ed., Vieweg+Teubner, 2011.
- Mitschke/Wallentowitz: Dynamik der Kraftfahrzeuge, 4. Auflage, Springer

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Engineering Knowledge Transfer Units to Increase Student´s Employability and Regional Development

Teaching “Basics in Vehicle Dynamics” 2

by Dr. Karl Reisinger

Lateral, Vertical Dynamics

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Lateral/Vertical Dynamics in Bachelors Program

- How does the car move due to steering angle? – Explanation using Single Track Model
  - w/o tire slip angle → Ackermann Kinematics
  - Vehicle states with tire slip angle
  - Basics to derive the equations of motion for linearized single track model
  - Principle behaviour read from ODE-System
- Understeer behaviour?
  - Testing, Goal, Understeer Gradient, Parameter's influence
- Two Track Model
  - Wheel centre speeds at each corner, discussion of single track model and two track model
  - Ackermann Steering, Role of differentials
- Vertical dynamics
  - Comfort, Quarter Vehicle Model, Road Description (Power density Spectrum)
- Simulation: Forward, Backward Sim., a view to veDYNA (TESIS)

Lateral/Vertical Dynamics in Masters Program

- Lateral wheel load transfer
  - Suspended, Non Suspended Masses
  - Influence of compliances and suspension geometry
- Vehicle's lateral potential
  - G-G-Diagram
  - Milliken Moments Diagram
- Ride – Suspension Spring and Damper
  - comfort, driving safety
- Simulation
  - AVL/VSM, a hands on course to get deep insight.

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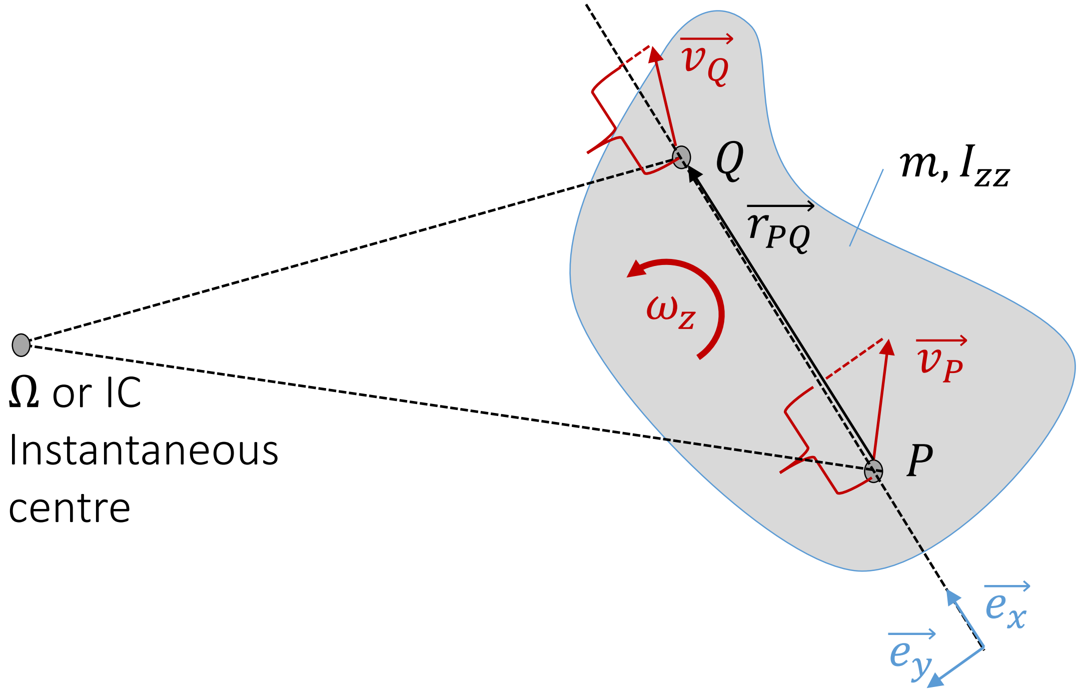
Kinematics in x-y-plane - Review

- Assume 2 fixed points at a body P, Q
- The body has a velocity $\overrightarrow{v_P}$ and rotates with $\vec{\omega} = \omega_z$
- We get the velocity in Q
  - $\overrightarrow{v_Q} = \overrightarrow{v_P} + \vec{\omega} \times \overrightarrow{r_{PQ}} = \overrightarrow{v_P} + \omega_z \cdot \overline{PQ} \cdot \overrightarrow{e_y}$
- velocity in direction of $\overline{PQ}$ doesn‘t change, if the body is rigid
- There is an virtual point, the Instantaneous center, which can be seen as a momentary hinge, the body turns about.
- Each velocity is perpendicular to the line ΩP and ΩQ



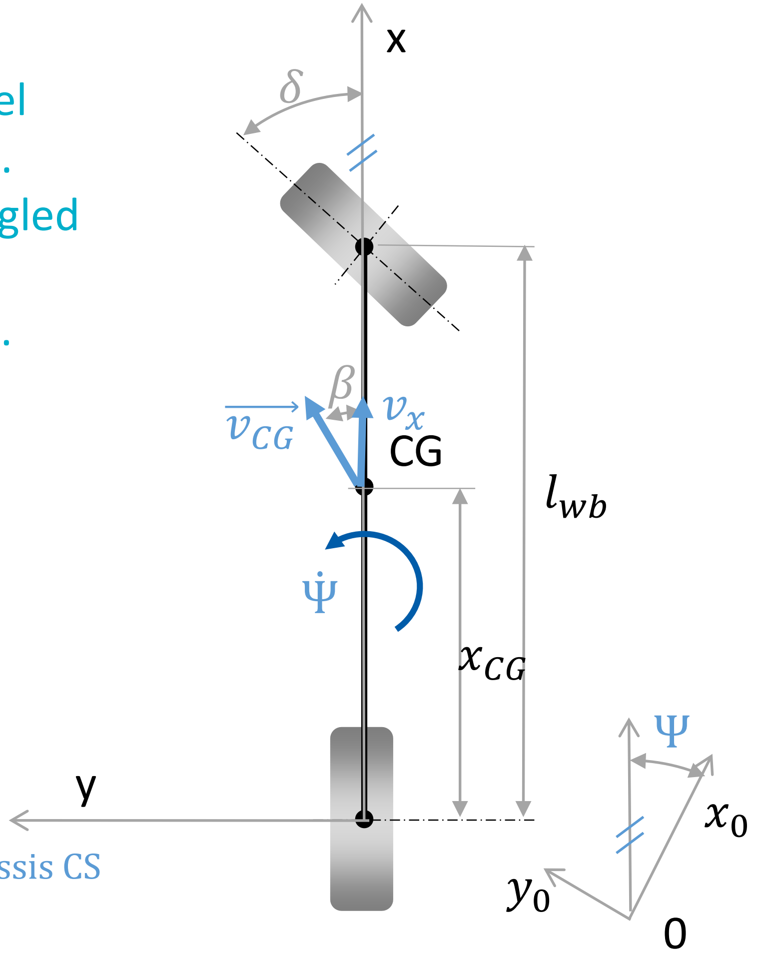
Task: Velocities w/o and with tire slip angle

- Given is a **Single Track Model driving a left** turn
  - representing the center of the car.
  - given are a steady state turn and
    - Longitudinal speed $v_x = \text{const}$, tire radius r_e
    - wheel base l_{wb}, the center of Gravity is x_{CG} in front of rear axle
    - we steer the front wheels by δ
    - Assume, we know the tire slip angles

  Scene A: Due to slow manoeuvring, we get low lat. acceleration and **neglectable tire slip angles**.

  Scene B: left turn, lateral acceleration produces a inertia force, which tries to move the car to right side. Thus we get **tire slip angles** pointing to the right side.
- Wanted: equations for scenes A and B for
  - Radius Ry, the y-distance of CG to IC
  - Radius R to the CG
  - Body slip angle β, the angle between $\overrightarrow{v_{CG}}$ and x-Axis
  - Yaw rate $\dot{\Psi}$
  - Velocity in CG
  - Wheel speeds front/rear neglecting long. slip.

Hints:
Sketch velocities in wheel centers and construct IC.
Then you‘ll find right-angled triangles.
Use Vehicle Coordinates.



x_0, y_0 .. Inertial CS, x , y .. Chassis CS
Ψ.. Heading Angle, Yaw Angle,
$\beta + \Psi$.. Course Angle
$\dot{\Psi} = \omega_z$.. Yaw Rate = rot. about z-axis

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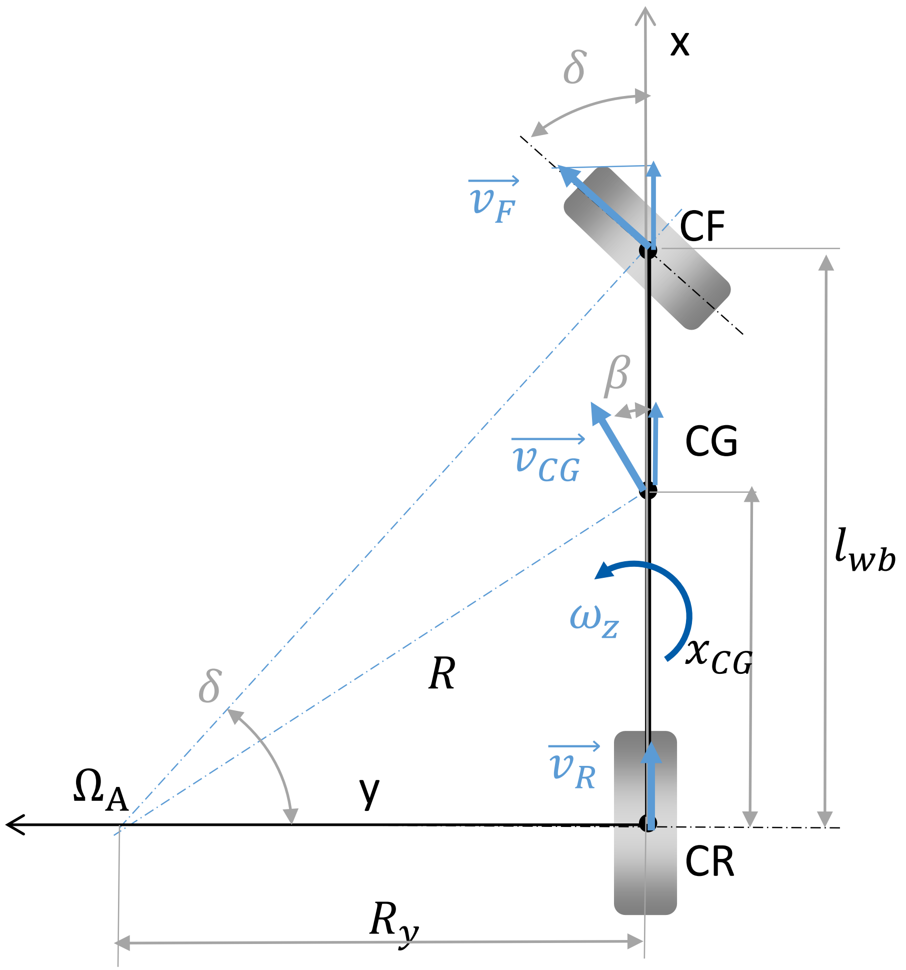
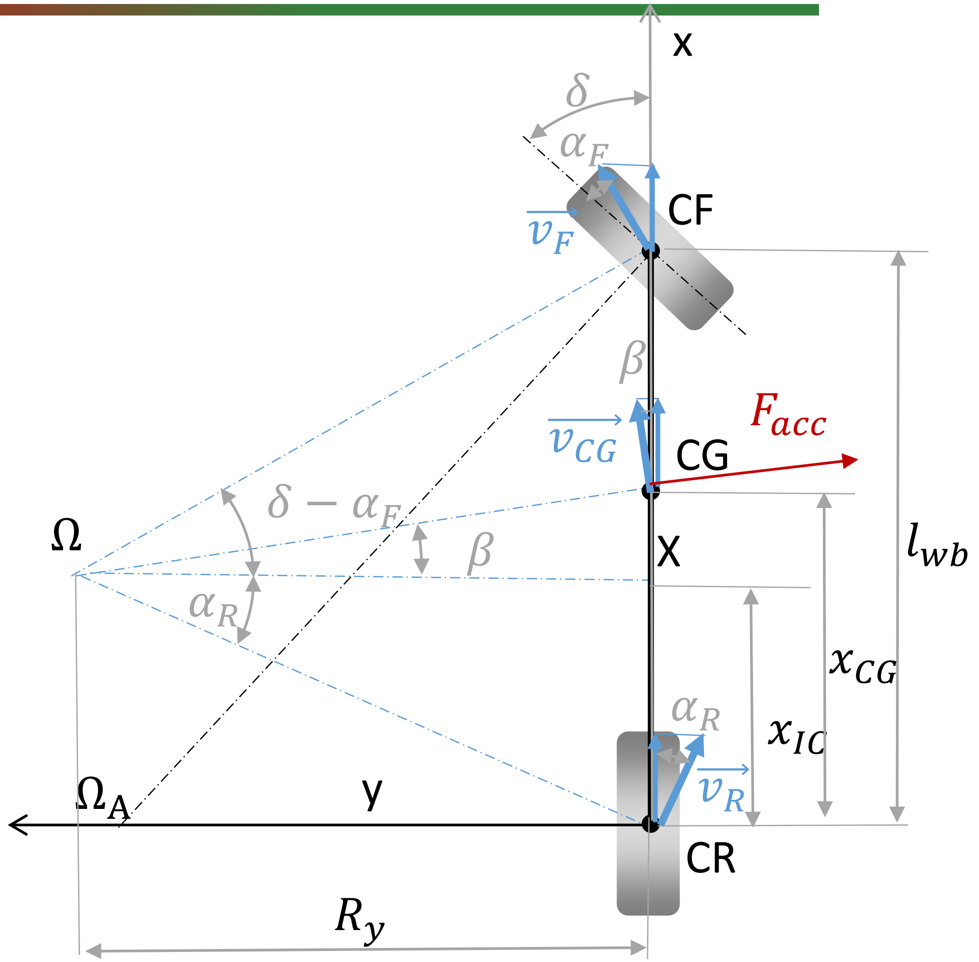
Scene A), no tire slip angle

- R_y: Triangle CF-CR-Ω_A
$$\tan(\delta) = \frac{l_{wb}}{R_y}$$
- R, β:Triangle CG-CR-Ω_A
$$R^2 = R_y^2 + x_{CG}^2, \qquad \tan(\beta) = \frac{x_{CG}}{R_y}$$
- Or $v_{x,CG} = v_x, v_{y,CG} = x_{CG} \cdot \omega_z, \tan(\beta) = \frac{v_{y,CG}}{v_{x,CG}}$
- $\dot{\Psi}, R$: Kinematics with $\dot{\Psi} = \omega_z$
$$v_x = R_y \cdot \dot{\Psi}, v_{CG} = R \cdot \dot{\Psi} = \frac{v_x}{\cos(\beta)}$$
- ω_w:In tire coordinates, no long. slip
$$v_{Fx}^T = \frac{v_x}{\cos(\delta)} = r_e \cdot \omega_{wF}, v_{Rx}^T = v_R = v_x = r_e \cdot \omega_{wR},$$



Scene B), with tire slip angle

- The tire follows the tire force → α_F, α_R points to the right in the left turn
- R_y, x_{IC}: Triangle X-CR-Ω + Triangle X-CF-Ω
  $$\tan(\alpha_R) = \frac{x_{IC}}{R_y}, \tan(\delta - \alpha_F) = \frac{l_{wb} - x_{IC}}{R_y}$$
- R, β:Triangle CG-X-Ω
  $$R^2 = R_y^2 + (x_{CG} - x_{IC})^2, \qquad \tan(\beta) = \frac{x_{CG} - x_{IC}}{R_y}$$
- Or $v_{x,CG} = v_x = v_R \cdot \cos(\alpha_R)$, $v_{y,CG} = -v_R \cdot \sin(\alpha_R) - x_{CG} \cdot \dot{\Psi}$,
  $$\tan(\beta) = \frac{v_{y,CG}}{v_{x,CG}}$$
- $\dot{\Psi}, R$: Kinematics with $\dot{\Psi} = \omega_z$
  $$v_x = R_y \cdot \dot{\Psi}, v = R \cdot \dot{\Psi} = \frac{v_x}{\cos(\beta)}$$
- ω_W:In tire coordinates, no long. slip
  $$v_F = \frac{v_x}{\cos(\delta)}, v_{Fx}^T = v_F \cdot \cos(\alpha_F) = r_e \cdot \omega_{WF},$$
  $$v_{Rx}^T = v_x = v_R \cdot \cos(\alpha_R) = r_e \cdot \omega_{WR}$$



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CG Acceleration

- P .. Origin of body fixed CS, this CS rotates with $\vec{\omega} = \dot{\Psi}$
- CG .. center of Gravity of body, Velocity $\overrightarrow{v_{CG}}$ points in dir. $\Psi + \beta$, rotates with $\dot{\Psi} + \dot{\beta}$ in z-direction

- Position $\overrightarrow{r_{CG}} = \overrightarrow{r_P} + \overrightarrow{r_{PCG}}$
- Velocity $\overrightarrow{v_{CG}} = \dot{\overrightarrow{rC_G}} = \dot{\overrightarrow{r_P}} + \dot{\overrightarrow{r_{PCG}}} + \vec{\omega} \times \overrightarrow{r_{PCG}},$

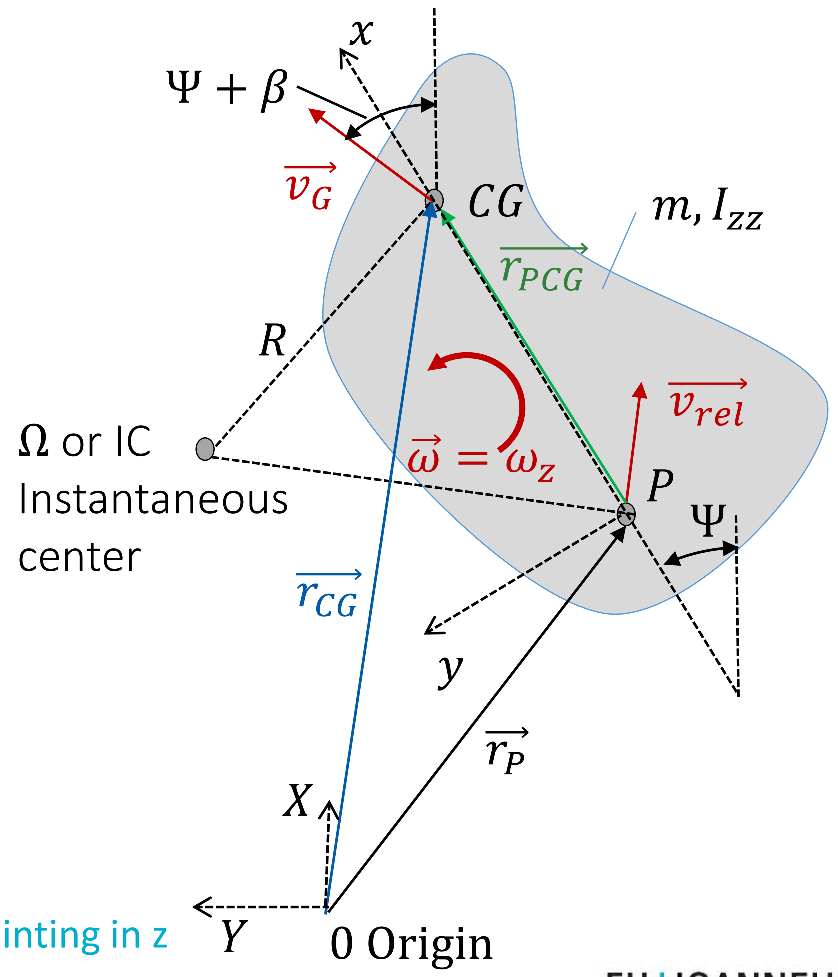
  with $\overrightarrow{v_{PCG}} = \left|\dot{\overrightarrow{r_{PCG}}}\right| \cdot \overrightarrow{e_x} = 0$: $\overrightarrow{v_{CG}} = \overrightarrow{v_P} + \vec{\omega} \times \overrightarrow{r_{PCG}},$
- Acceleration

$$\overrightarrow{a_{CG}} = \overrightarrow{a_P} + \overrightarrow{a_{PGG}} + \dot{\vec{\omega}} \times \overrightarrow{r_{PCG}} + 2\,\vec{\omega} \times \overrightarrow{v_{PCG}} + \vec{\omega} \times \overrightarrow{v_{CG}}$$
$$\overrightarrow{a_{CG}} = \overrightarrow{a_{CS}} + \overrightarrow{a_{rel}} + \overrightarrow{a_{Euler}} + \overrightarrow{a_{Coriolis}} + \overrightarrow{a_{Zentripedal}}$$

With $\overrightarrow{a_{PGG}} = \overrightarrow{v_{PCG}} = 0$, rigidly connected
$$\overrightarrow{a_{CG}} = \overrightarrow{a_P} + \dot{\vec{\omega}} \times \overrightarrow{r_{PCG}} + \vec{\omega} \times \overrightarrow{v_{CG}},$$

Generally $\dot{\vec{\omega}} = \ddot{\Psi} \neq 0$, $\overrightarrow{a_C}$ points in x and y
- CG acceleration doesn't point to IC
- Longitudinal acceleration: $\boldsymbol{a_{CGx} \neq a_{Px}}$
- Centripetal acceleration: $\boldsymbol{a_{CGy}} = \frac{\mathbf{v_{CG}^2}}{\boldsymbol{\rho}} = \mathbf{v_{CG}} \cdot (\dot{\boldsymbol{\Psi}} + \dot{\boldsymbol{\beta}})$ ... see also Mitschke Wallentowitz S. 552
- $\boldsymbol{\rho \neq R}$ ... ρ curvature radius of CG path, R ... distance CG to IC

Steady State $\dot{\vec{\omega}} = \ddot{\Psi} = 0, v_x = const, \dot{\beta} = 0, \delta = const$ ... stabilized circular driving
- CG acceleration points to IC
- $\boldsymbol{\rho = R}$ ... ρ curvature radius of CG path, R ... distance CG to IC
- Longitudinal acceleration: $a_{CGx} = 0$
- Centripetal acceleration: $a_{CGy} = \frac{\mathrm{v_{CG}^2}}{R} = \mathrm{v_{CG}} \cdot \dot{\Psi} = R \cdot \dot{\Psi}$

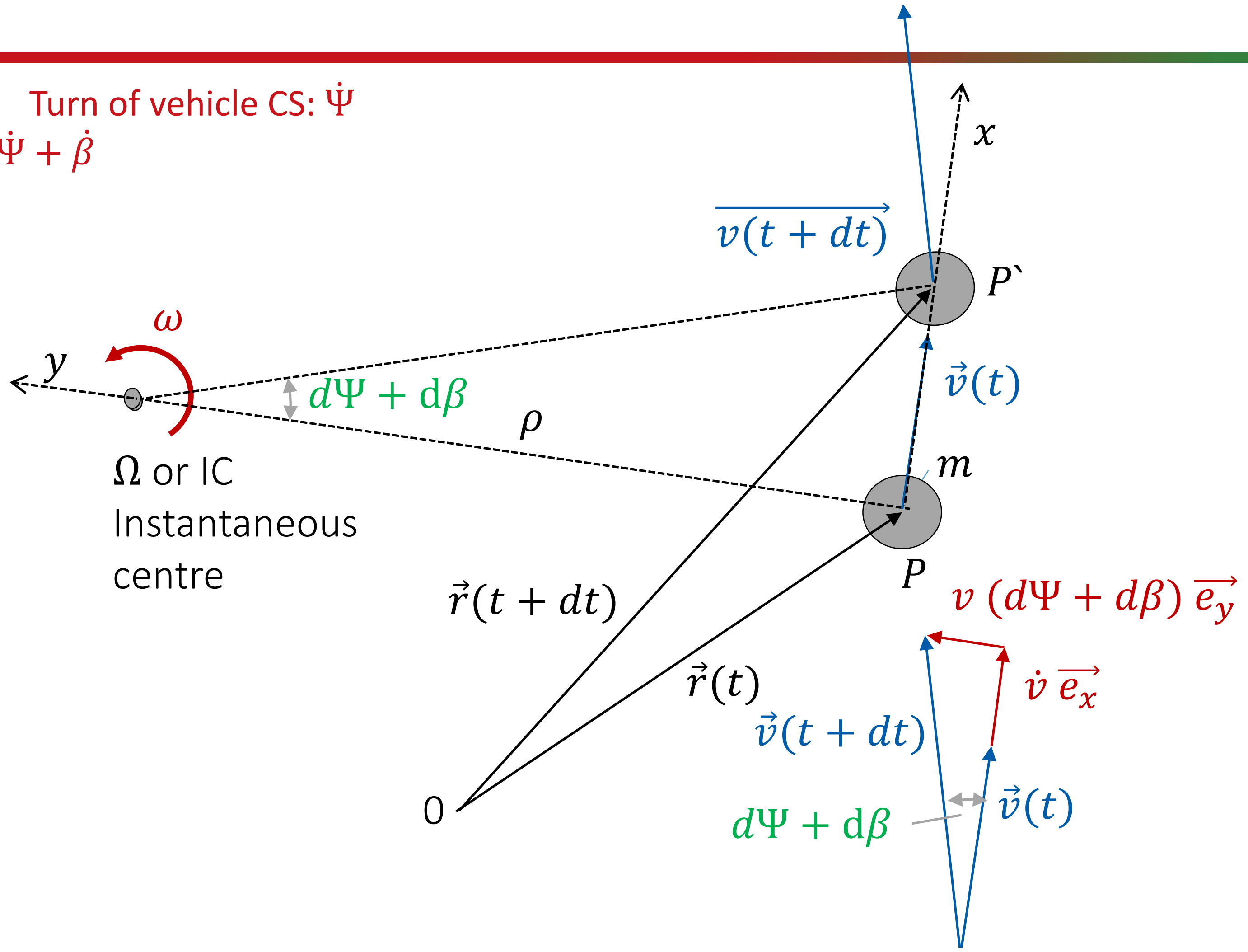


Please differentiate! Turn of vehicle: $\vec{\omega} = \dot{\Psi}$ pointing in z
Turn of CG velocity: $\dot{\Psi} + \dot{\beta}$ gives centripetal acc.

Acceleration

Please differentiate! Turn of vehicle CS: $\dot{\Psi}$

Turn of CG velocity: $\dot{\Psi} + \dot{\beta}$



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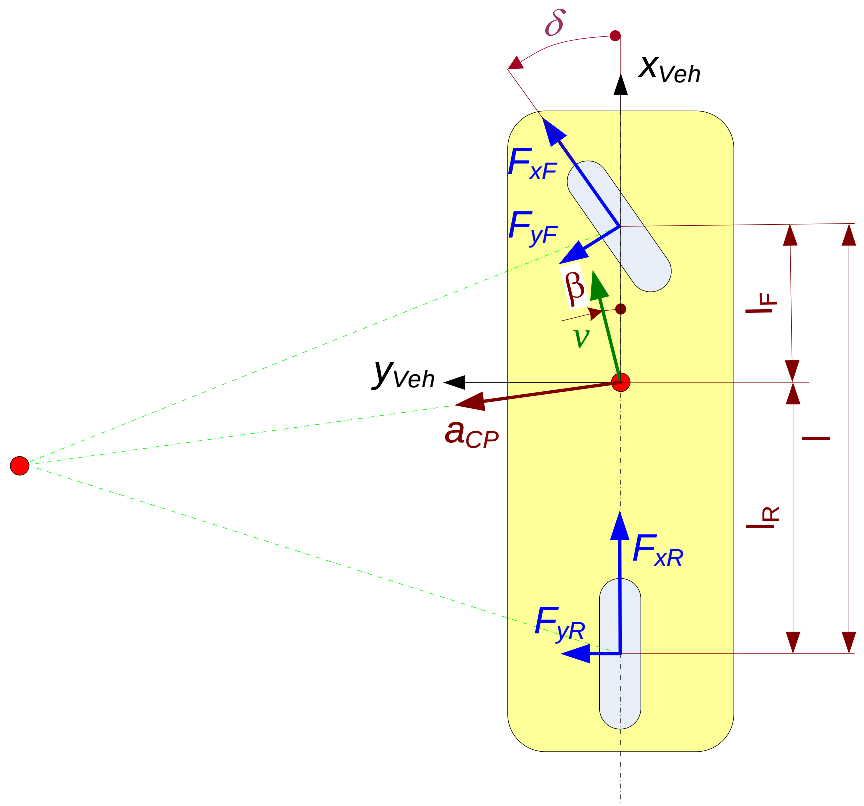
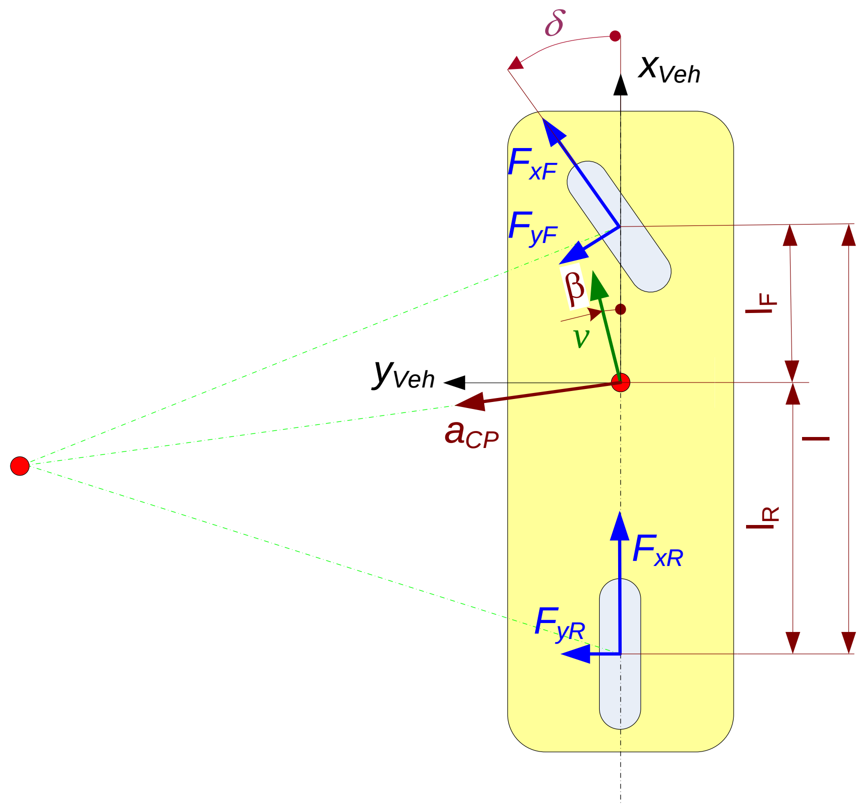
Kinetics1: Principle of linear momentum



- Position $\vec{r}(t) = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$
- $m\,\vec{a} = \vec{F}_F + \vec{F}_R$
- $m \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = \bar{A} \cdot \begin{pmatrix} F_{xF,T} \\ F_{yF,T} \\ 0 \end{pmatrix} + \begin{pmatrix} F_{xR} \\ F_{yR} \\ 0 \end{pmatrix}$
- $\bar{A} = \begin{pmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

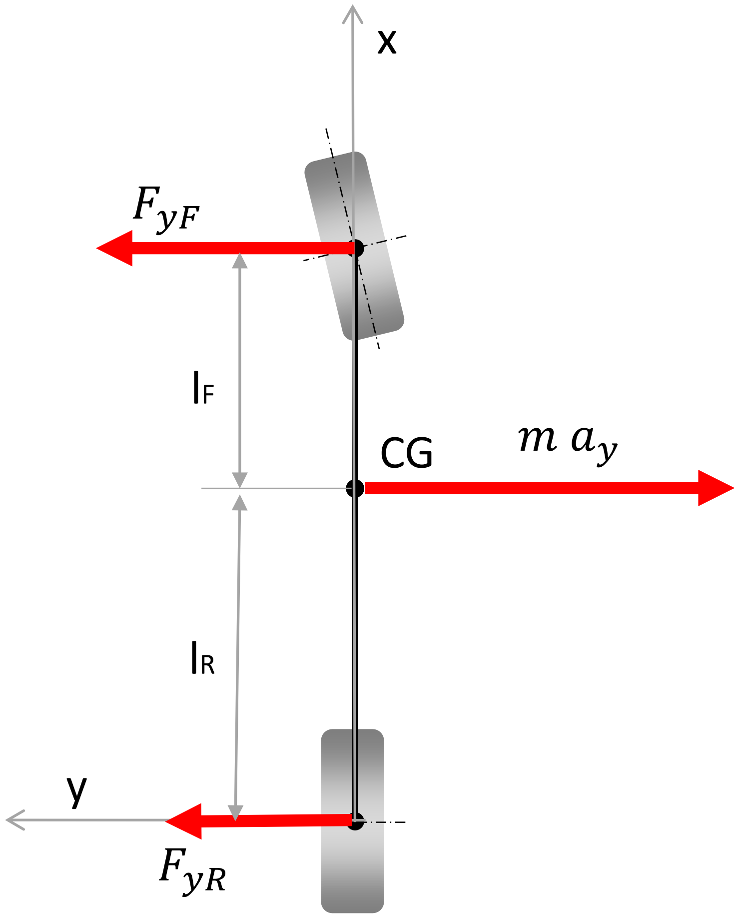
Kinetics 2: Principle of angular momentum



- $I_{zz} \cdot \begin{pmatrix} 0 \\ 0 \\ \ddot{\Psi} \end{pmatrix} = \begin{pmatrix} l_f \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} F_{xF,T} \\ F_{yF,T} \\ 0 \end{pmatrix} + \begin{pmatrix} -l_R \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} F_{xR} \\ F_{yR} \\ 0 \end{pmatrix}$
- $I_{zz}\,\ddot{\Psi} = (F_{xF,T}\sin\delta + F_{yF,T}\cos\delta) \cdot l_f - F_{yR} \cdot l_R$

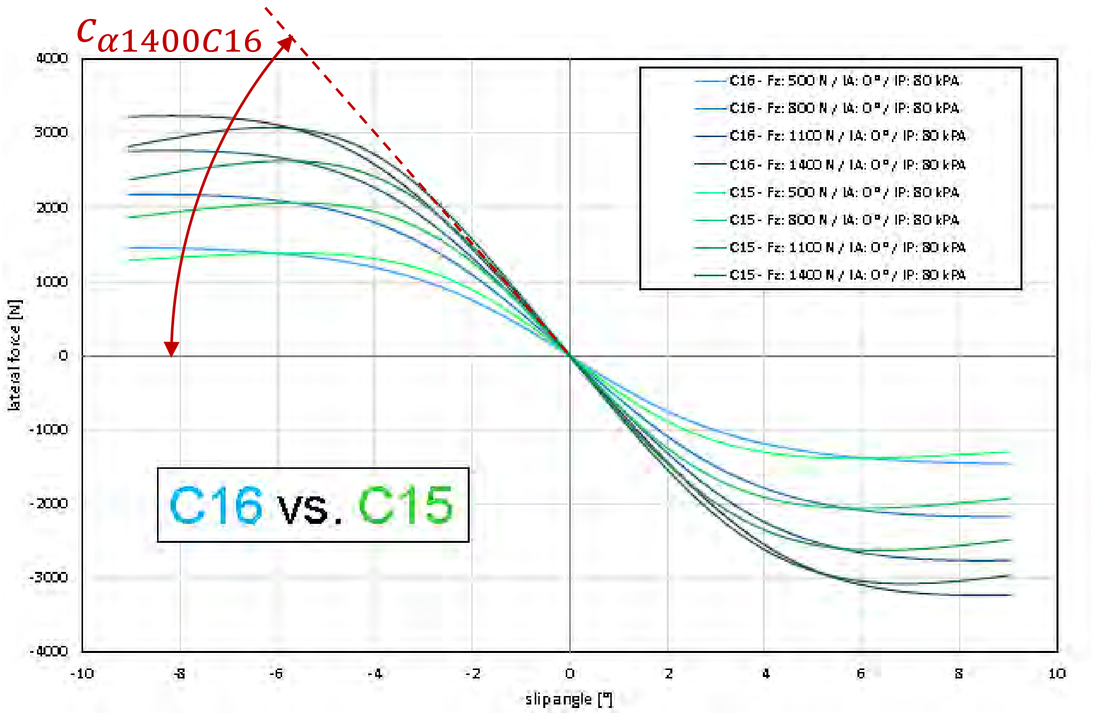
Kinetics for small angles, $\boldsymbol{a_x = 0}$

- $m \cdot a_x = F_{xF} + F_{xR}$
- $m \cdot a_y = F_{yF} + F_{yR}$
- $I_{zz} \ddot{\Psi} = F_{yF,T} \cdot l_f - F_{yR} \cdot l_R$



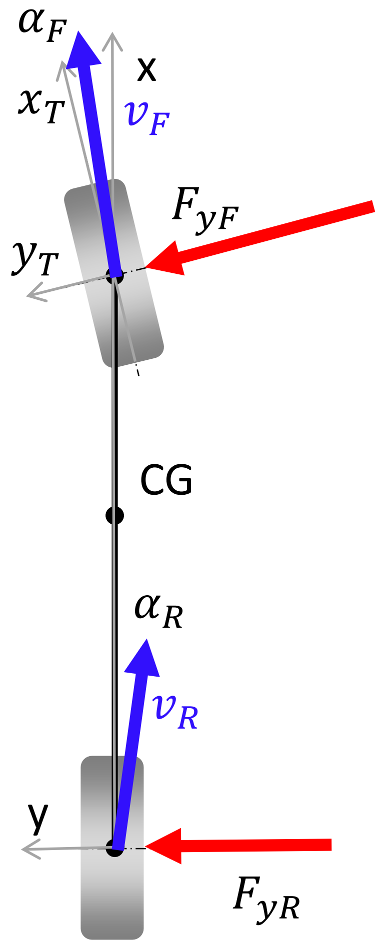
Linearized tire Model

Example: $c_{\alpha 1400C16}$ lin. approach for tire C16, Fz=1400N



Source: Continental
C15, C16 2 types of Formula S racing tires

- $F_{yF}^{T} = \alpha_F \cdot c'_{\alpha F}$
- $F_{yR} = \alpha_R \cdot c_{\alpha R}{}'$
- $c_\alpha{}'$ … cornering stiffness including compliance (=suspension weakness)
- $c_\alpha = f(F_z)$, use correct F_z
  - Remember: we have 2 wheels per axle to transmit wheel load and side force.



Equations of Motion

Don‘t memorize this result, watch it in books.
Please remember the shape and the influencing parameters

- 2 ODE‘s of 1. Order in β und $\dot{\Psi}$
- $\dot{\beta} = -\frac{c_{\alpha F}+c_{\alpha R}}{\mathrm{m\,v}} \cdot \beta + \left(\frac{c_{\alpha R} \cdot l_R - c_{\alpha F} \cdot l_F}{m\,v^2} - 1\right) \cdot \dot{\Psi} + \frac{c_{\alpha F}}{m\,v} \cdot \delta$
- $\ddot{\Psi} = -\frac{c_{\alpha F} \cdot l_R - c_{\alpha R} \cdot l_F}{\mathrm{I_{ZZ}}} \cdot \beta - \frac{c_{\alpha R} \cdot l_R^2 - c_{\alpha F} \cdot l_F^2}{I_{ZZ}\,v} \cdot \dot{\Psi} + \frac{c_{\alpha F}}{I_{ZZ}} \cdot \delta$

- $\dot{\beta} = f_1 \cdot \beta + f_2 \cdot \Psi + \dot{f}_3 \cdot \delta$
- $\ddot{\Psi} = f_4 \cdot \beta - f_5 \cdot \dot{\Psi} + f_6 \cdot \delta$

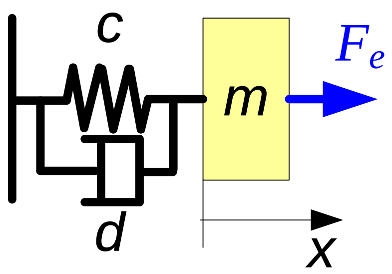
- $f_i = f\left(c_{\alpha F,R}, l_{F,R}, m, I_{ZZ}, v\right) \neq f(t)$

Remember how we got it!

- Kinematic Constraints
- Kinetics
- Tire

Force Excited Single Mass Vibrator

- Linear Momentum $m\,\ddot{x} = -F_{cd} + F_e$
- Link Equation $F_{cd} = +c\,x + d\,\dot{x}$
- ODE $\ddot{x} = -\frac{c}{m}x - \frac{d}{m}\dot{x} + \frac{F_e}{m}$



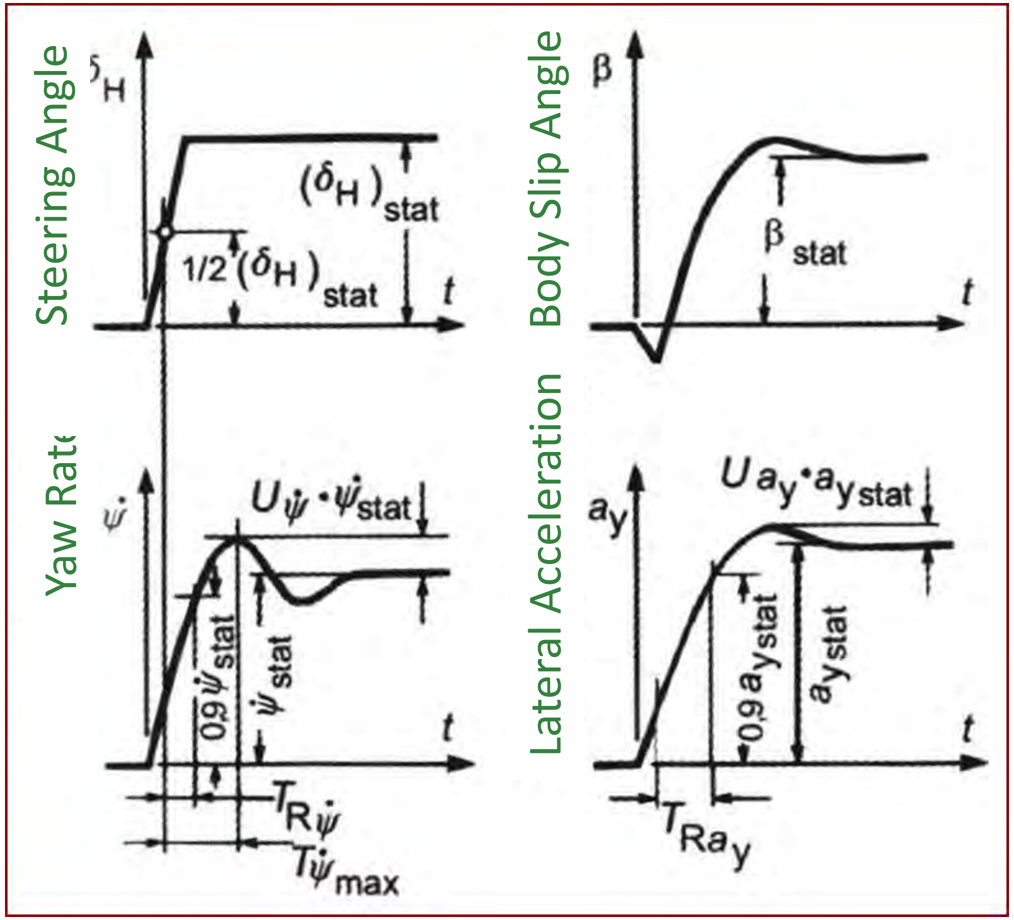
- Substitution $z_1 = x, \dot{z}_1 = \dot{x}$
- $z_2 = \dot{x}, \dot{z}_2 = \ddot{x}$
- Result $\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{d}{m} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} \cdot F_e$

behavior of a car is described by

- 2 ODE‘s of 1st order in β and $\dot{\Psi}$
  - $\dot{\beta} = -\frac{c_{\alpha F}+c_{\alpha R}}{\mathrm{m\ v}} \cdot \beta + \left(\frac{c_{\alpha R} \cdot l_R - c_{\alpha F} \cdot l_F}{m\, v^2} - 1\right) \cdot \dot{\Psi} + \frac{c_{\alpha F}}{m\, v} \cdot \delta$
  - $\ddot{\Psi} = -\frac{c_{\alpha F} \cdot l_R - c_{\alpha R} \cdot l_F}{\mathrm{I_{ZZ}}} \cdot \beta - \frac{c_{\alpha R} \cdot l_R^2 - c_{\alpha F} \cdot l_F^2}{I_{ZZ}\, v} \cdot \dot{\Psi} + \frac{c_{\alpha F}}{I_{ZZ}} \cdot \delta$
- State Space Representation
  - $\begin{pmatrix} \dot{\beta} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \beta \\ \omega \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \cdot \delta$
- Compare to single mass vibrator with force exciting
  - $\begin{pmatrix} \dot{z_1} \\ \dot{z_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{d}{m} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} \cdot F_e$

- Highly damped, oscillatory stable or instable system
- Parameters
  - Cornering stiffness‘
  - Position of CG
  - Mass, Inertia I_{zz}
  - Speed

Transient Testing



[Heißing02]

- We watch at constant speed $v_x = const$
  - Body Slip Angle $\beta(t)$
  - Yaw Rate $\dot{\Psi}(t)$
  - Lateral Acceleration $a_y(t)$
- Steering wheel step response
  - Open Loop Control
  - Measure time to reach 90% of steady state value
  - Overshoot U
  - steady state value
- Sine Input
  - Increase steering input frequency slowly but continuously
  - Measure responses $\beta(t)$, $\dot{\Psi}(t)$, $a_y(t)$
  - Use Fast Fourier Transformation to generate a Bode-Diagram
- Result: Yaw-Damping, Yaw-Eigenfrequency

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Sine Steering

https://www.youtube.com/watch?v=_RrhctXIJKU&t=8s

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Steady State Circular Driving

UNITED

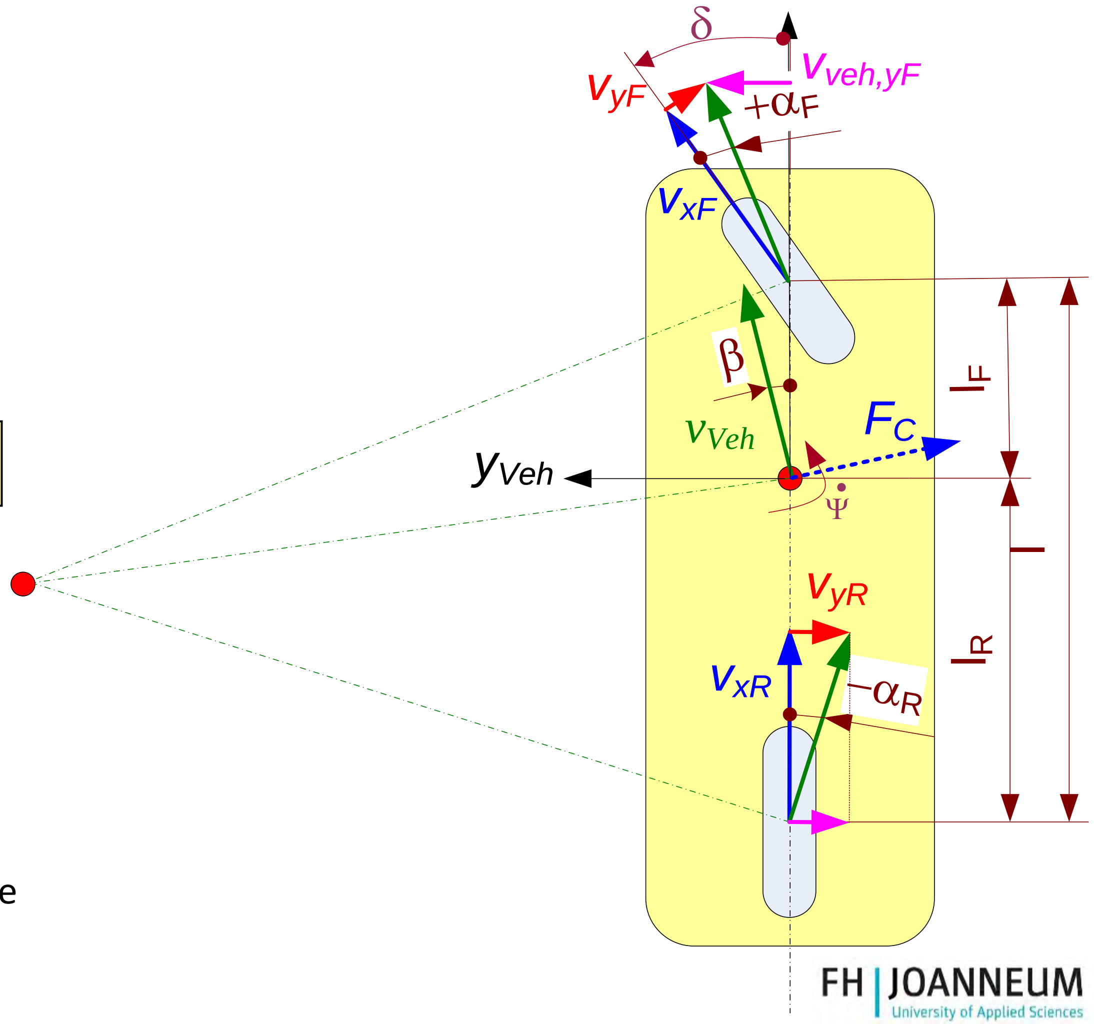
- Needed steering wheel input to get radius R

$$\delta \cong \frac{l}{R} + (\alpha_F - \alpha_R) = \delta_A + (\alpha_F - \alpha_R)$$

- δ must be increased to compensate front tire slip angle,
- δ must be decreased to compensate rear tire slip angle!
- Using single track model:

$$\delta \cong \underbrace{\frac{l}{R}}_{\text{Ackermann}} + \underbrace{m\frac{v_x^2}{R_y}}_{F_y = m \cdot a_y} \underbrace{\left(\frac{1}{c'_{\alpha F}} \cdot \frac{l_R}{l} - \frac{1}{c'_{\alpha R}} \cdot \frac{l_F}{l}\right)}_{\text{Correction}}$$

- Lateral Acceleration is increased by speed or Radius: $a_y = \frac{v_x^2}{R_y}$
- If we have to
- steer more, $(\alpha_F > \alpha_R)$ → UNDERSTEERING
- steer less, $(\alpha_F < \alpha_R)$ → OVERSTEERING
- Otherwise: NEUTRAL
- Cornering Stiffness $c'_{\alpha F/R}$ including tire and suspension compliance



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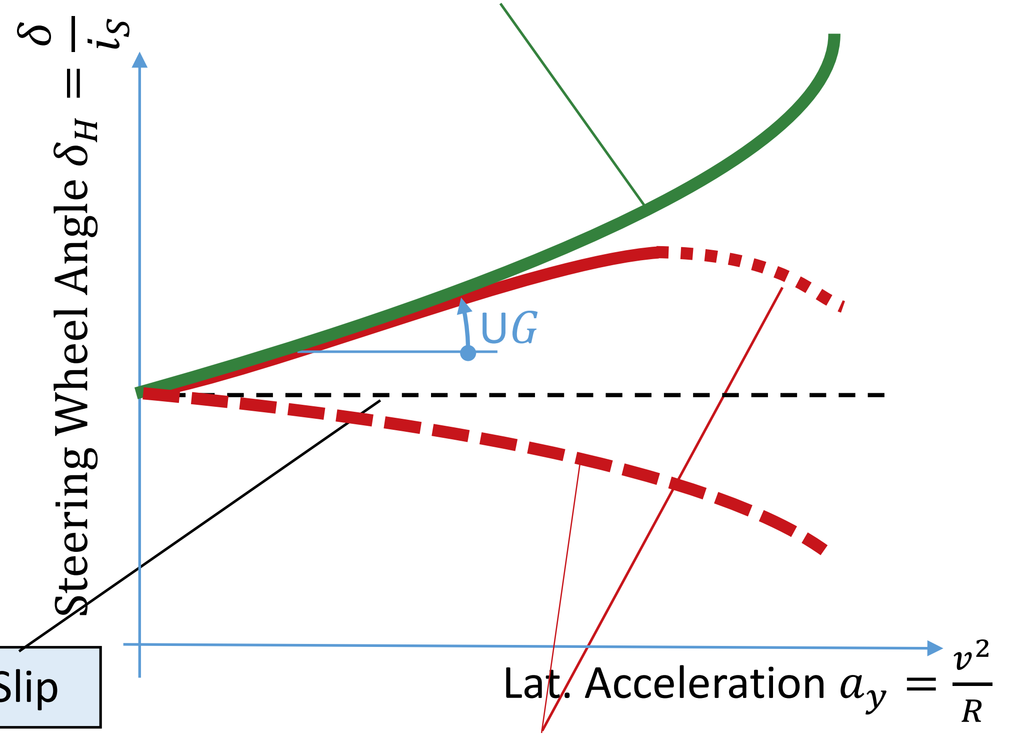
Steady State Circular Driving Test with constant Radius

- Steady State: slow or step by step acceleration at test circle.
  - Steer to stay on test circle (R=40m, 100m)
  - No load change, no long. accel.
  - Measure $\delta_H \; vs. a_y$
  - $\delta_H = \frac{\delta}{i_S}$, i_S .. Steering gear ratio
  - $UG = \frac{d(\alpha_F - \alpha_R)}{da_y} = \left(\frac{1}{c'_{\alpha F}} \cdot \frac{l_R}{l} - \frac{1}{c'_{\alpha R}} \cdot \frac{l_F}{l}\right)$ .. Understeer Gradient
- Understeer
  - That is, what we want, $\frac{d\delta}{da_y} > 0$
- Oversteer
  - That is, we have to avoid, also in racing cars!
  - Don't mix with power induced oversteer, inr rear wheel drive

We have to increase δ with increased a_y to stay on track.
If we do nothing, we get a stable wider circle, having less accel.

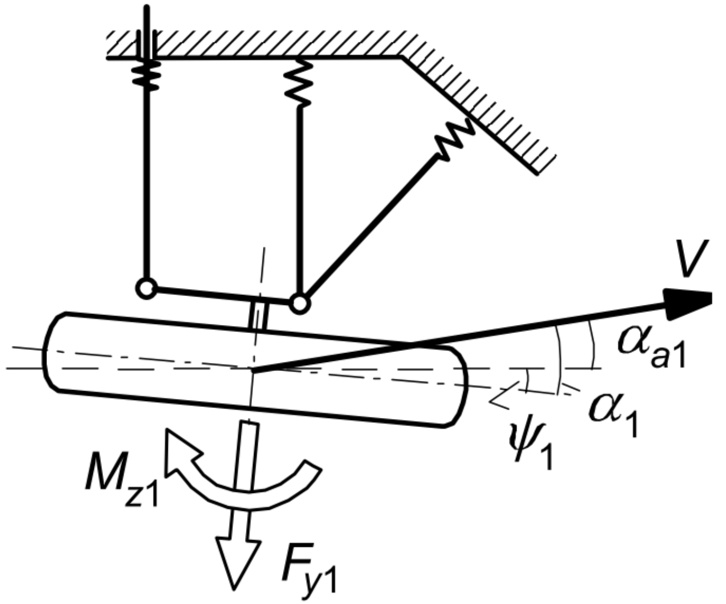


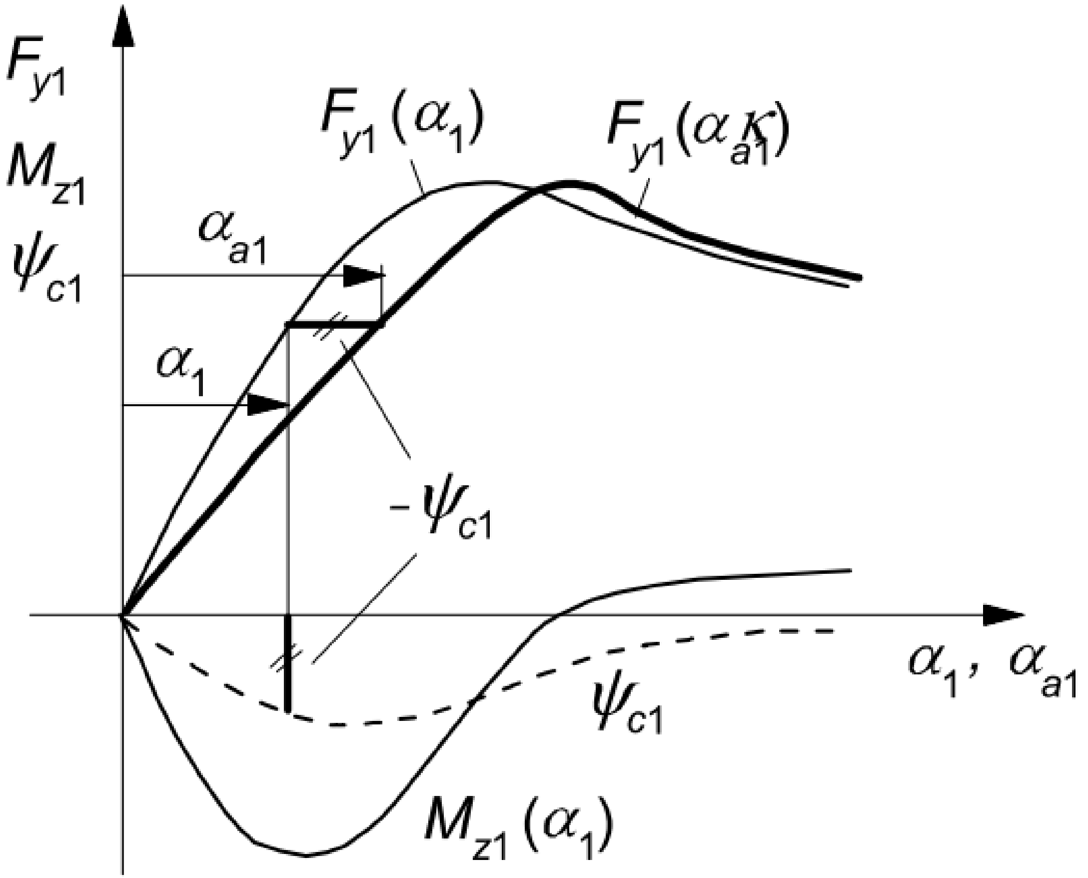
Counter-steering necessary to stay on any circle, otherwise skidding!

Test Facility
[www.magnasteyr.com]

Influence of suspension and steering compliance

- „compliance=1/stiffness“

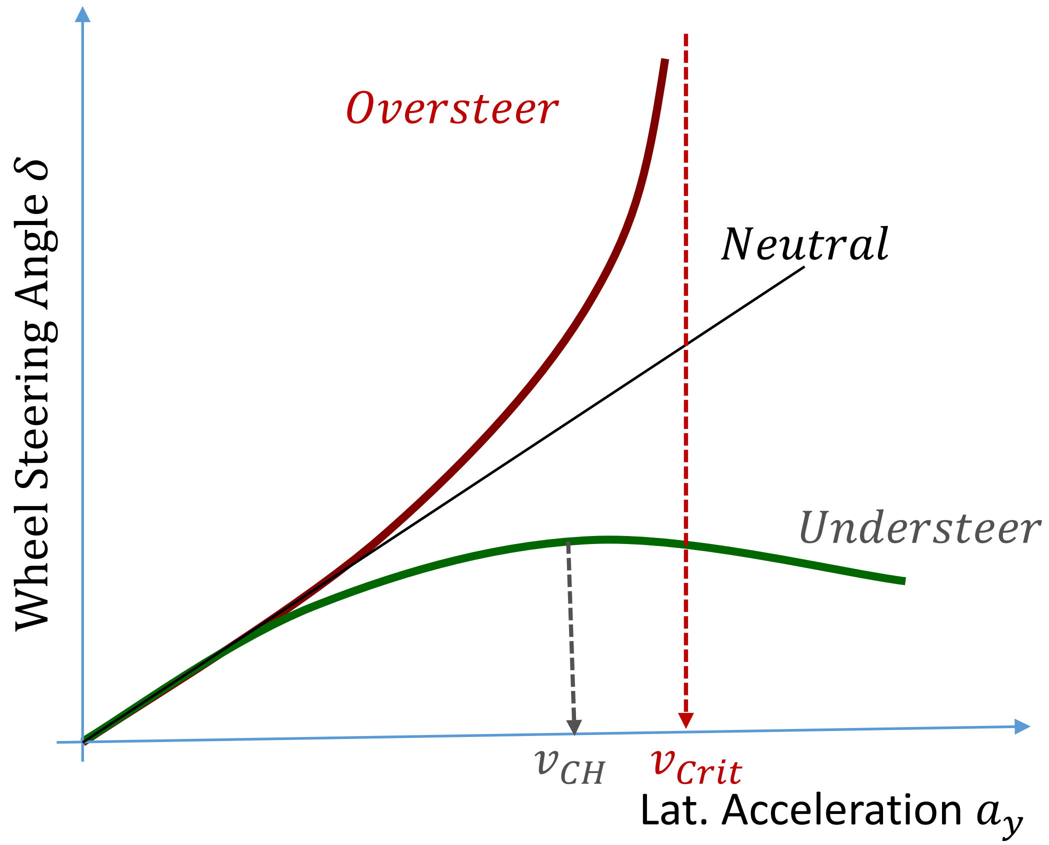




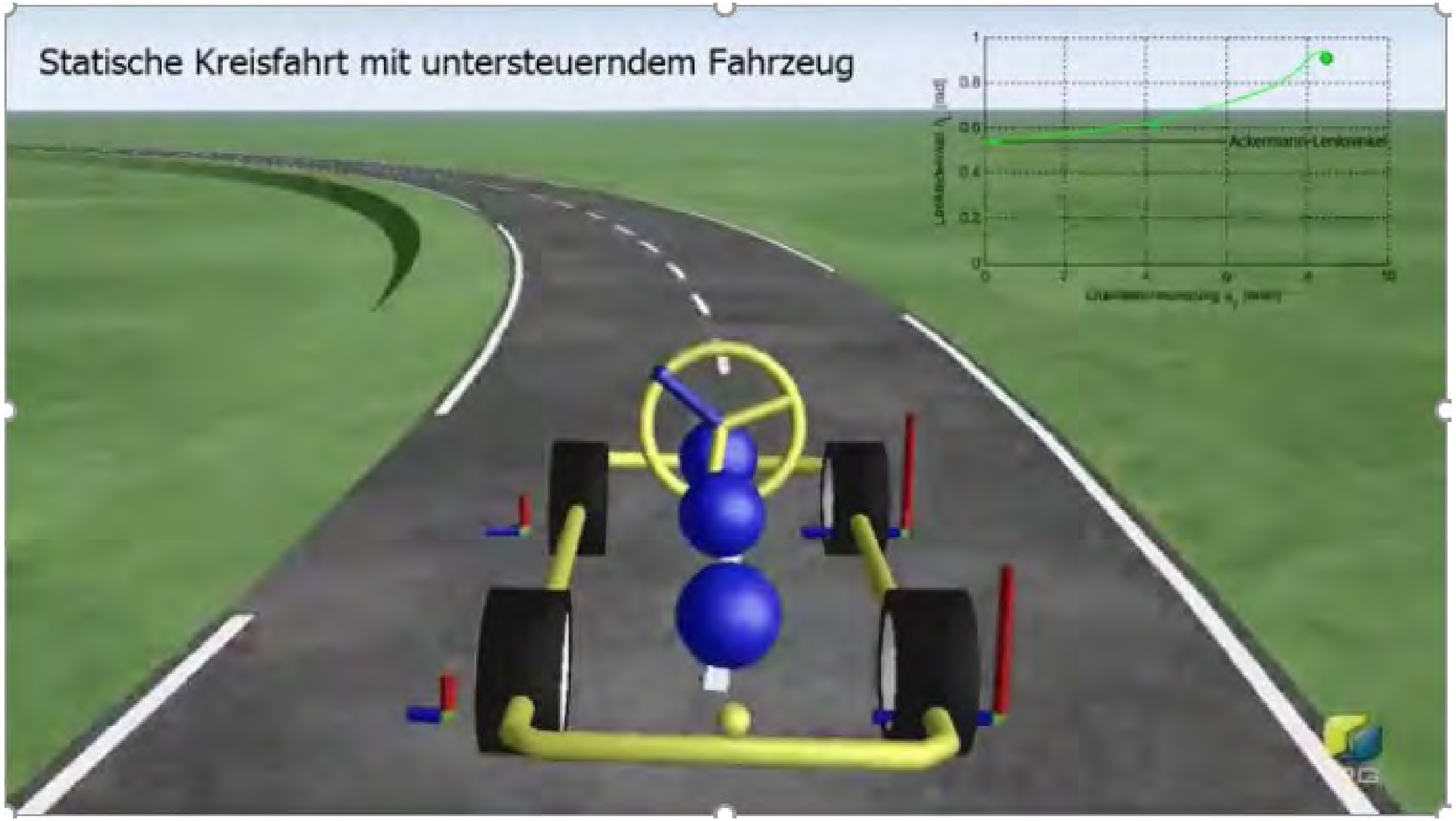
[Pacejka H.: Tyre and Vehicle Dynamics, Elsevier Amsterdam et. al. 2006]

Yaw Intensification

- Steady State Yaw Intensification
- $\frac{\dot{\Psi}}{\delta} = \frac{v}{l_{wb} + \frac{\partial \delta}{\partial a_y} \cdot v^2}$
    - Use ODE's for single track model to derive
- Oversteering Cars
    - Have a critical speed v_{crit}
- Understeering Cars
    - Have a characteristic speed v_{char}, the speed with best response to steering input
    - 65 km/h < v_{ch} < 100km/h

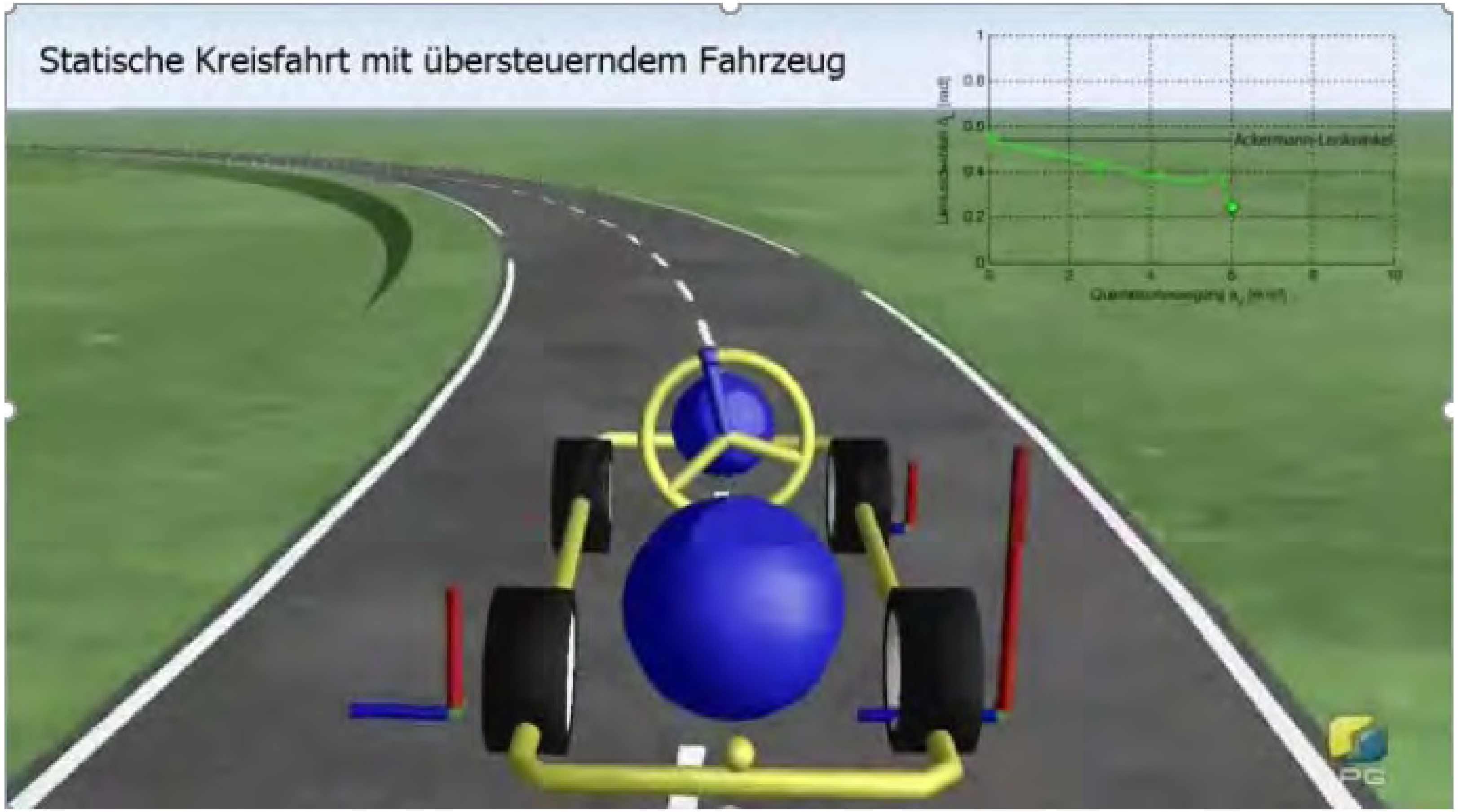


Understeer – Watch the steering wheel



https://www.youtube.com/watch?v=pWKCilizzkU&list=TLPQMjcwMTIwMjDU2sYsGCVbJA&index=2

Oversteer – Watch the steering wheel



https://www.youtube.com/watch?v=pWKCilizzkU

Power induced oversteering

https://www.youtube.com/watch?v=shwgNV36xFA&t=4s

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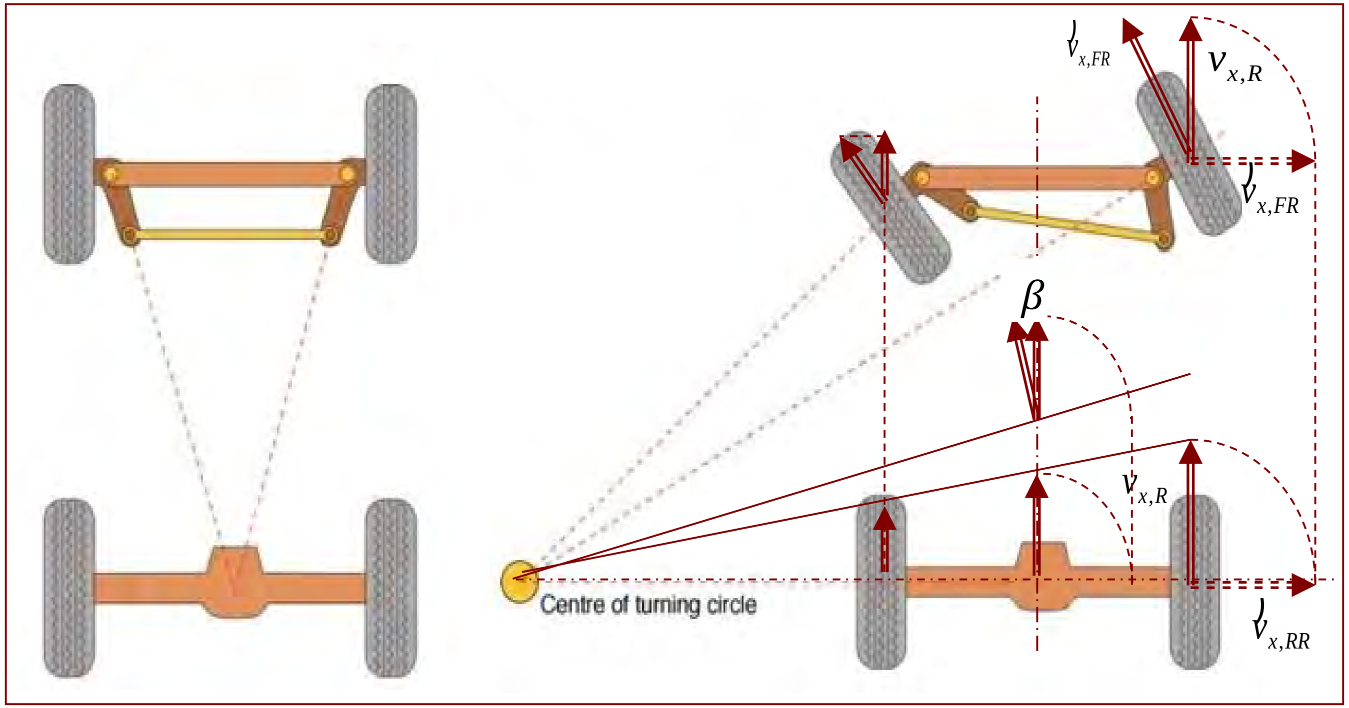
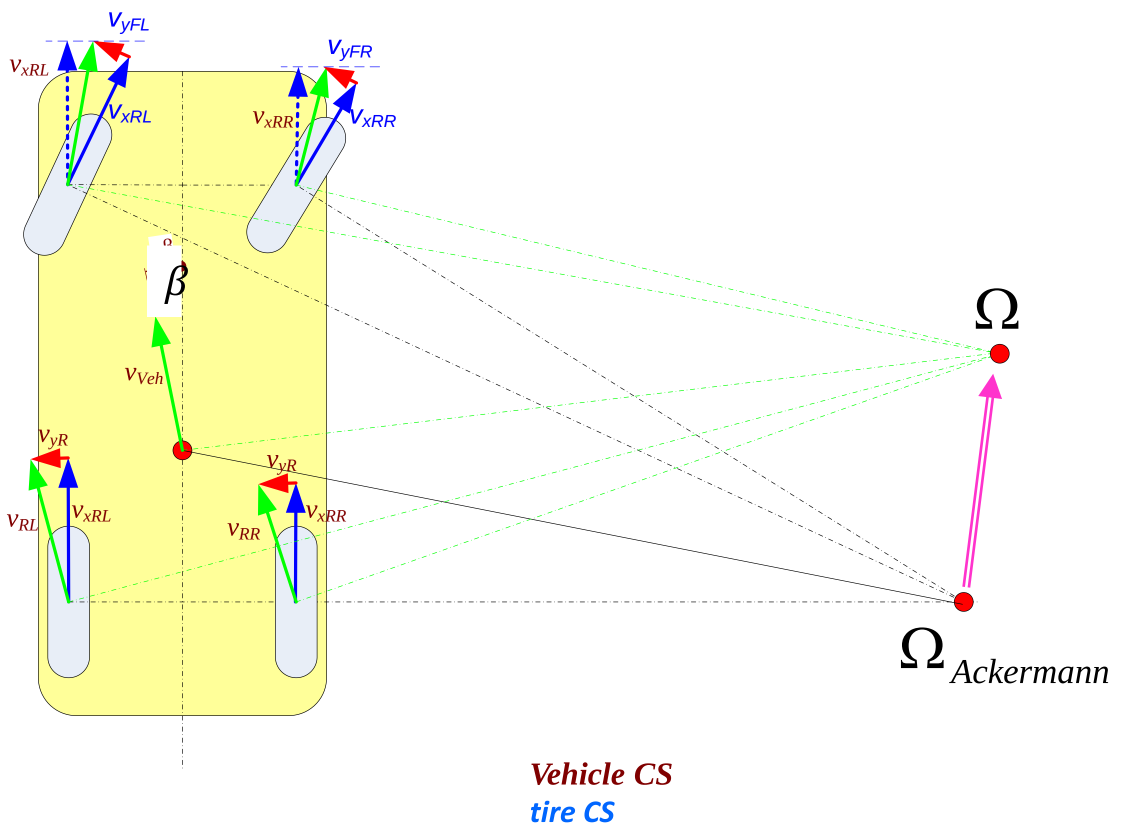
2-track model in the x-y-plane – low speed
$\boldsymbol{a_y \ll 1 \;\rightarrow F_y \cong 0, \alpha \cong 0}$

- Rudolph Ackermann
    - 1764–1834
  - Steering Trapezoid
  - velocities for no tire slip
  - 100% Ackermann:
    steering bars cross at rear axle.
- Wheel speeds
  - $v = r_e \omega$
  - higher at outer side
  - higher at front
  - Mean front speeds > mean rear speeds
- Typically solution:
  - Max. wheel steering angle is defined by space and drive joint.
  - less steering at inner wheel to decrease turning circle.



[http://en.wikipedia.org/wiki/Image:Ackermann_New.jpg]

2-track model in the x-y-plane with tire slip



- Instantaneous center
  - Moves to front
  - At wider radii (if understeering)
- Wheel speeds
  - High difference left/right $v_L > v_R$ → Axle Differential
  - Less difference of mean values front and rear compared to Ackermann $\frac{v_{xFL}+v_{xFR}}{2} \approx \frac{v_{xRL}+v_{xRR}}{2}$ → center Differential can be locked at higher speeds.

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Compensation via road

-

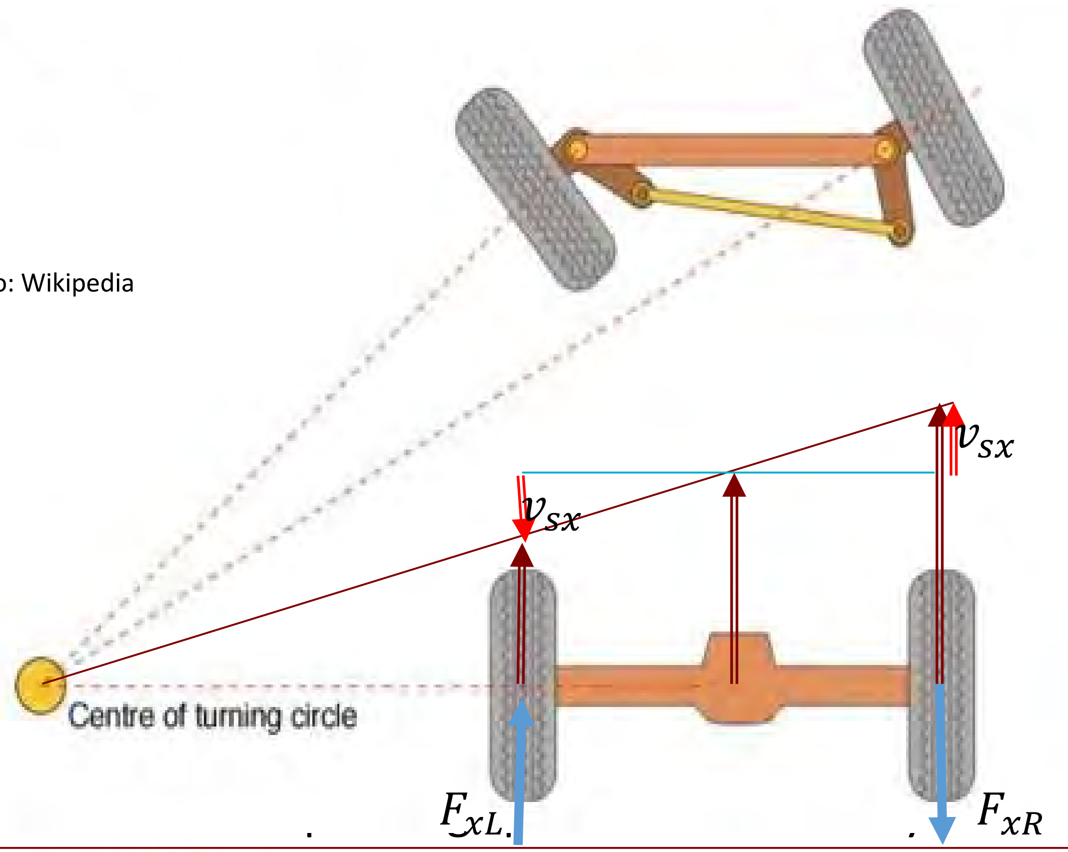
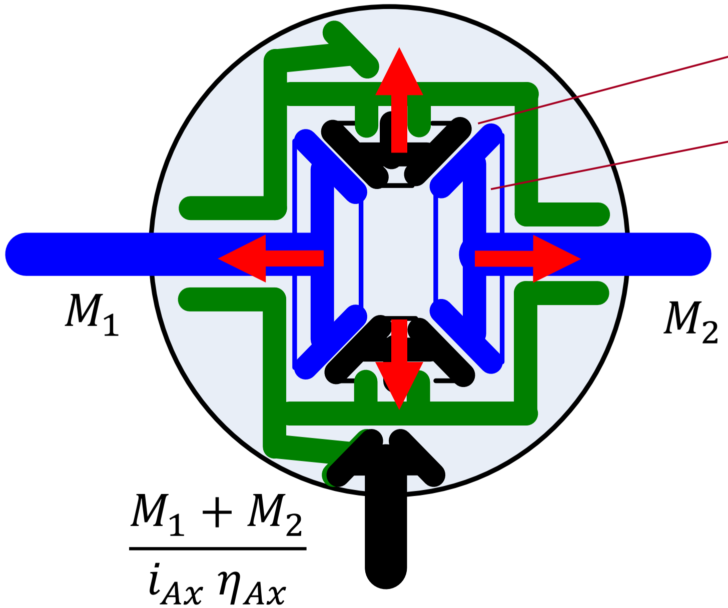


Foto: Wikipedia

Differential



Axial, radial: slide bearings

Radial: slide bearings or roller bearings
Axial: slide bearings, roller bearings
or friction surface to increase friction

- Equalizes speed differences
- Splits torque
  - Due to radii of toothed wheels on both outputs
    - Axle: same radius, split 50:50
- Torque depending friction
  - We like it to increase traction on µ-Split (ice on one side of car)

- TORque SENsitive
  - Locking Ratio (Sperrgrad) (EU)
  - $S = \frac{|M_1 - M_2|}{|M_1 + M_2|}$, $0 < S \leq 1$, typ. 10% - 15%
  - Torque Bias Ratio (US)
  - $TBR = \frac{\max(M_1, M_2)}{\min(M_1, M_2)}$, typ: $1 < TBR < 10$

What does AWD?

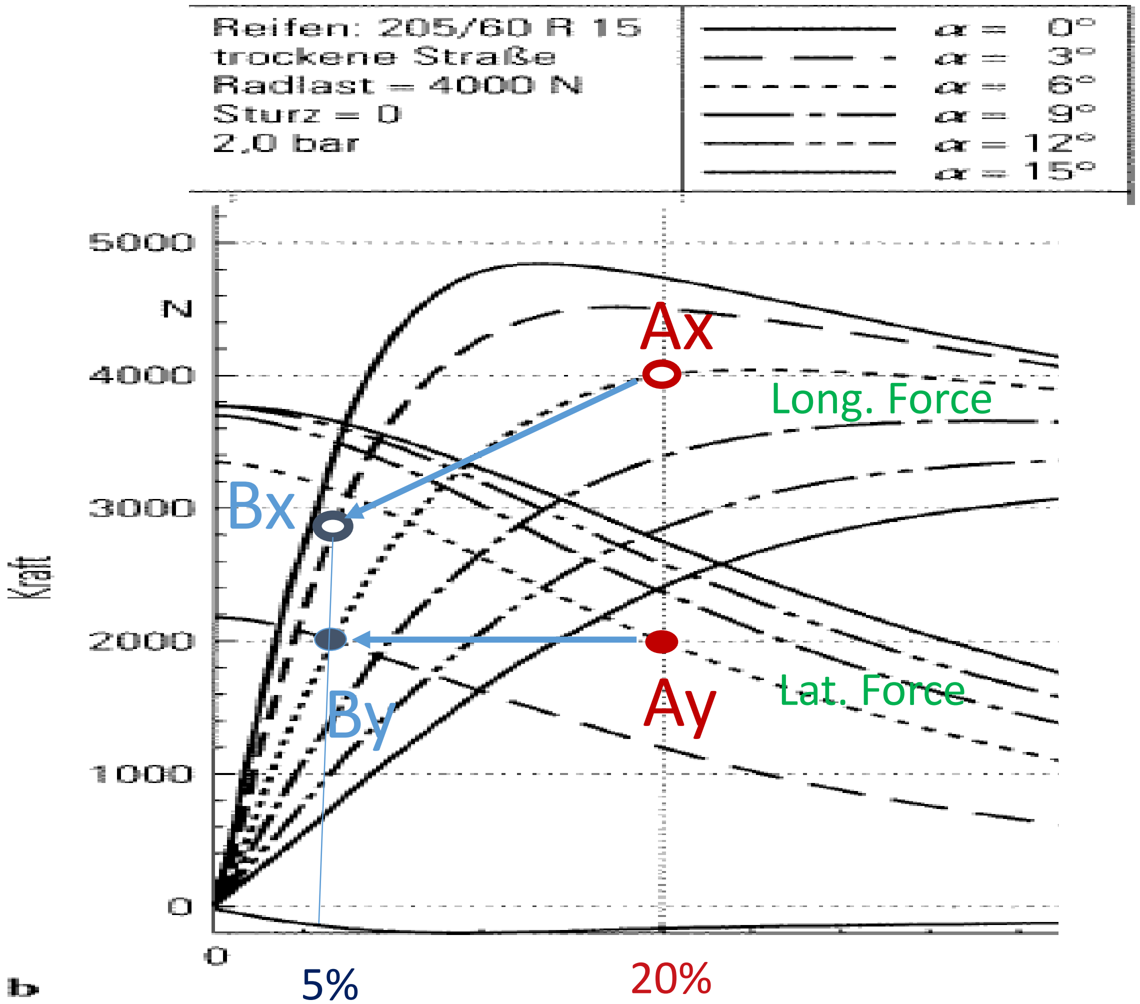


Bild schl Traction and Lateral Force vs. slip for different α

Forces, slip	A: RWD	B: AWD
F_x	4000 N	2800 N
F_y aus a_y	2000 N	2000 N
s_x	20 %	5 %
α	6°	3°

Reduce longitudinal force
→ less long. slip
→ tire can transfer more lat. force
→ less lateral slip at same side force
AWD influences front/rear tyre slip angle and Understeer Gradient.

(https://en.wikipedia.org/wiki/Weight_transfer)

Weight Transfer:
A motorcyclist performing a stoppie

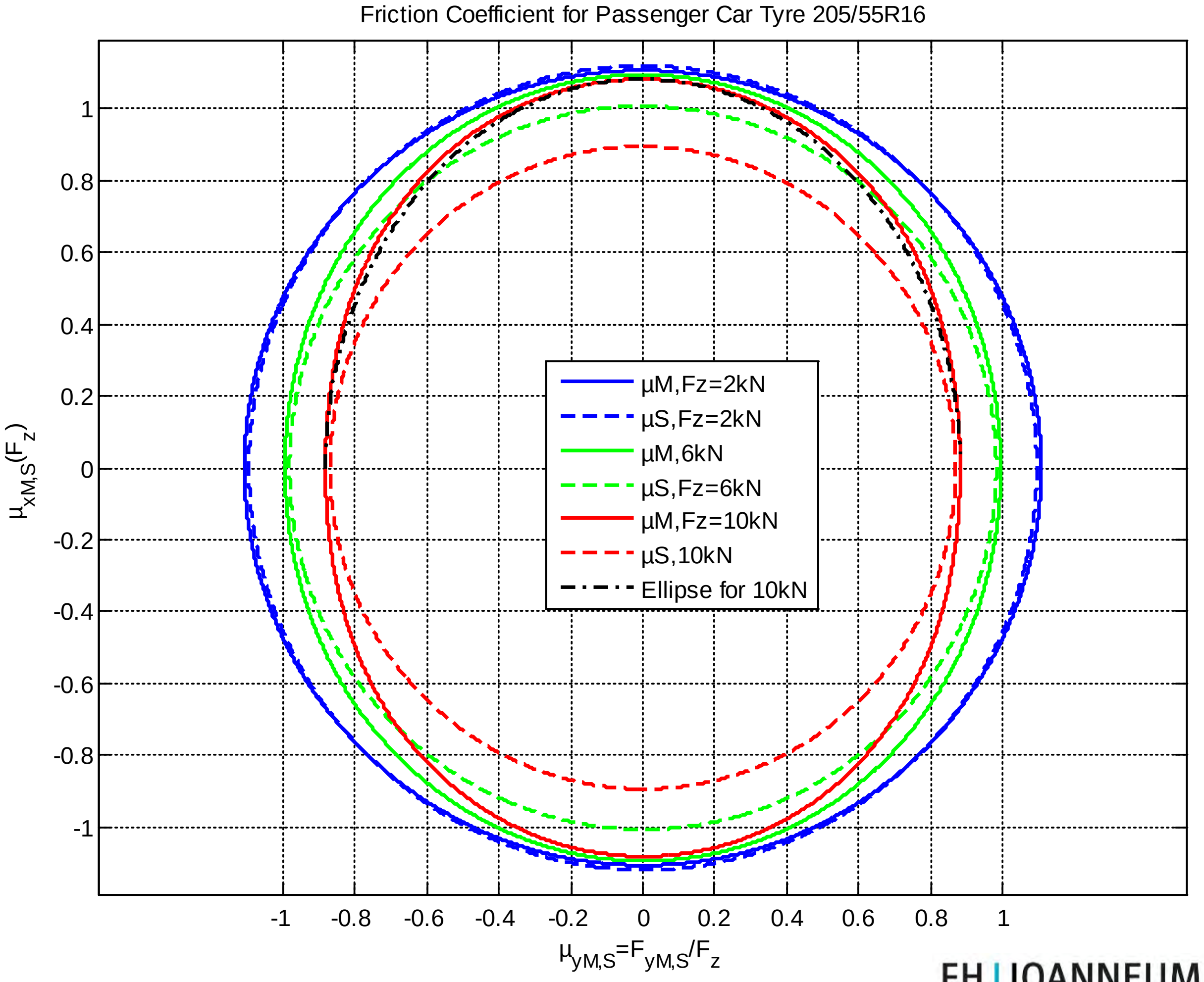
Co-funded by the Erasmus+ Programme of the European Union

Dr. Karl Reisinger

Friction Circe depending on $\mu_{m,s}(F_z)$

- **We loose µ with increasing load, especially in lateral direction!**
- **driving a turn:**
  - **"You loose more in the inner side, than you gain in the outer side"**
- **We must know the wheel load distribution left/right to know the axle's side force potential and side slip.**

$\mu M = \frac{F^M}{F_z}$ .. max. friction
μS.. sliding friction coefficient
passenger car tyre, result of TM-Easy



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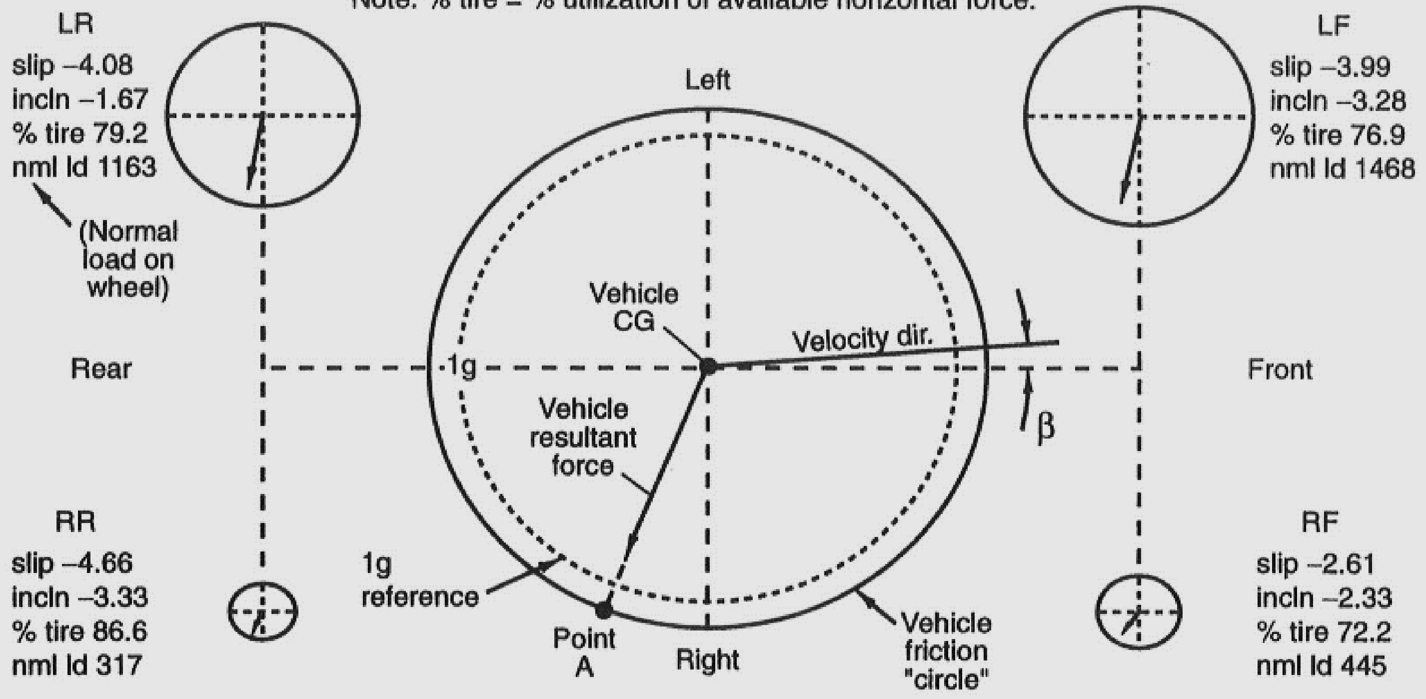
Tyre forces at each wheel while cornering and g-g-diagram

Moderate braking dropped throttle

Note: % tire = % utilization of available horizontal force.



delsw (δ)	beta (β)	A_X	A_Y	C_N	N	V	r	R	phi (ϕ)
deg.	deg.	g	g	–	lb. - ft.	mph	deg. / sec.	ft.	deg.
13.06	–3.761	–0.318	0.757	0.0	0.0	60.00	15.63	323.4	–3.100

Rouelle C.: Advanced Vehicle Dynamics Applied to Race Car Design & Development, www.optimum.com

Wheel Loads @ Three-Wheeler

- $\sum M_y = 0$
  - for Front / Rear axle
- $\sum M_x = 0$
  - roll moment is distributed by front axle only

Use static equations to determine the wheel loads.

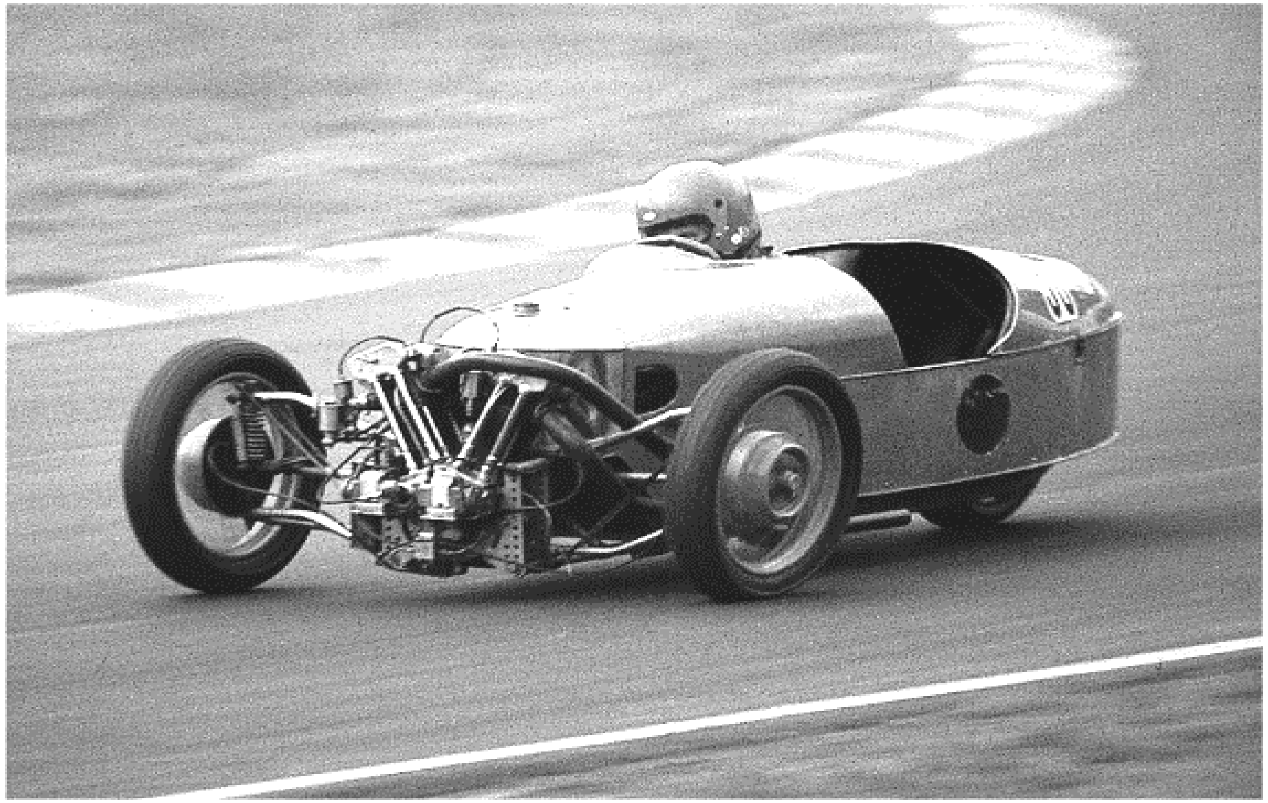


Figure 1: Morgan Three-Wheeler, MY 1932 (Wikipedia)

Tractors have a hinge joint at front axle

Quelle: Deutsche Fotothek

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Lateral Total Weight Transfer: Front+Rear in sum

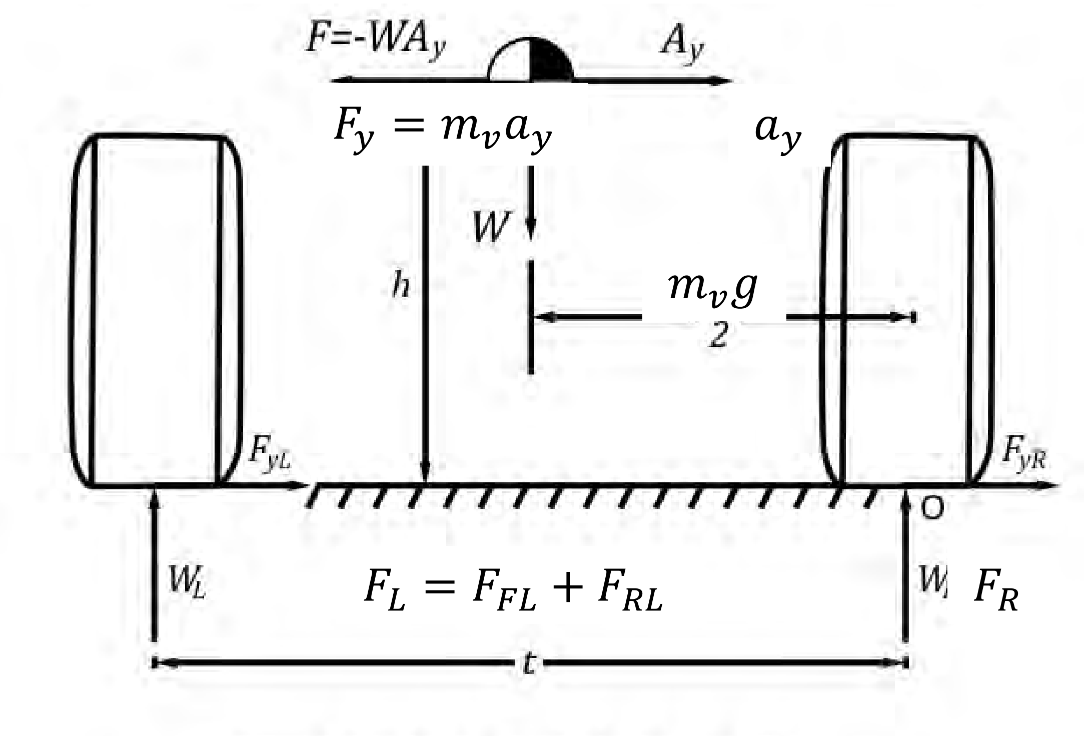

Figure 1. Free body diagram of a car, rear view.

(http://racingcardynamics.com/weight-transfer/)

- $F_{L,R} = m_v \left[\frac{g}{2} \pm \frac{h_{CG}}{t} a_y\right]$
- tip over if $F_R < 0$
- Influence of suspension in steady state: NONE! (except camber)
- The sum of the weight transfer front and right depends on the ratio CG-height above road over track width, $\frac{h_{CG}}{t}$
- But we can choose the ratio of weight transfer front over rear to influence the vehicle dynamics.

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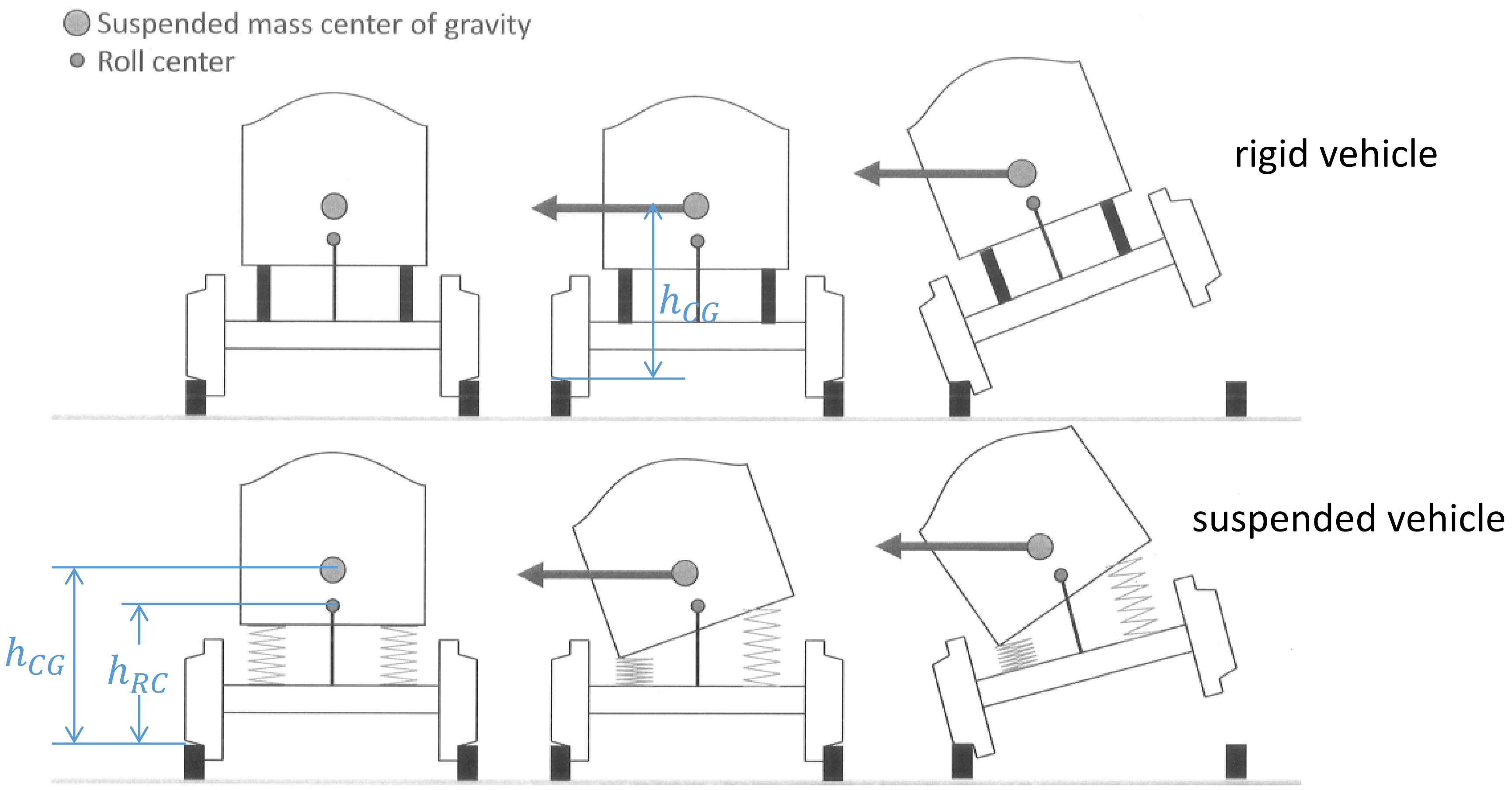
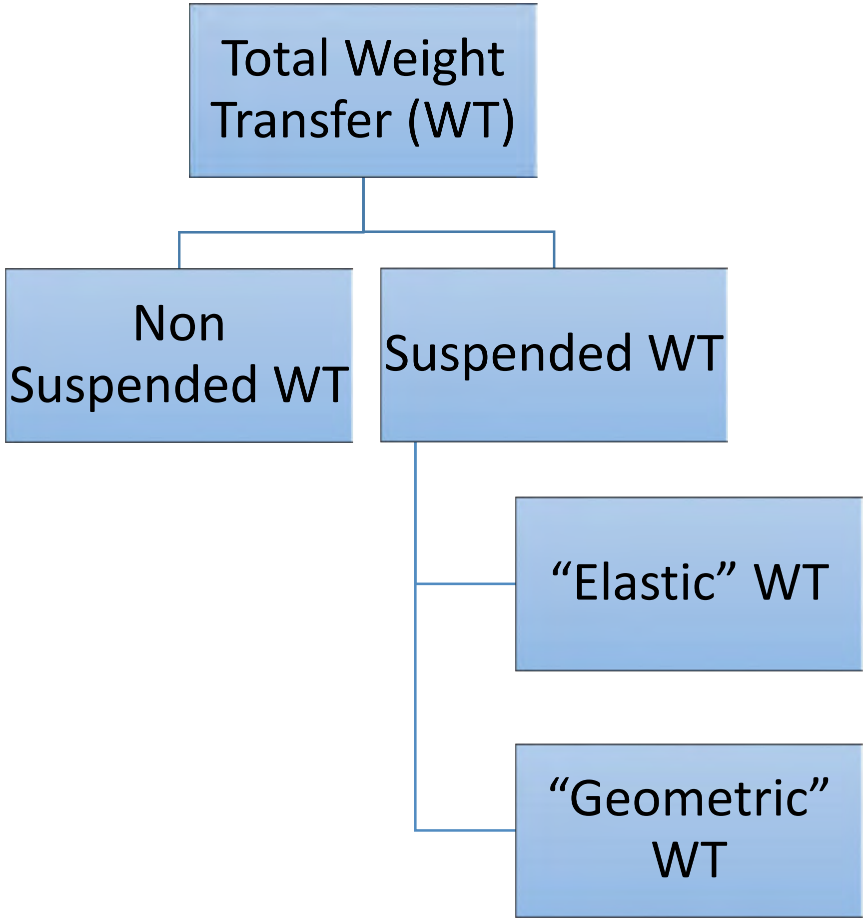


WT of suspended mass and non suspended mass

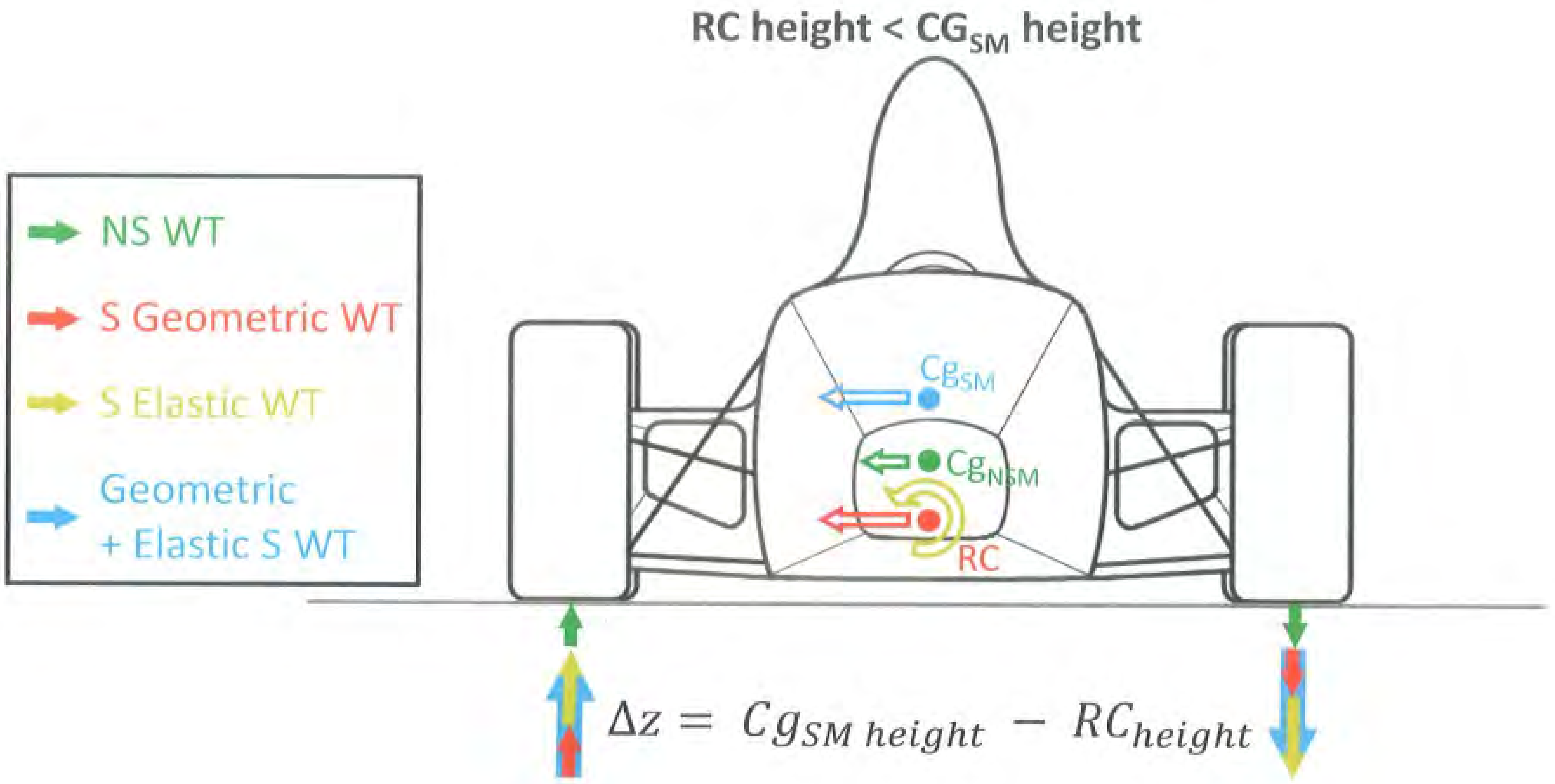
$\boldsymbol{m_{SM}} \cdot \boldsymbol{a_y} \cdot \boldsymbol{h_{CG}}$ makes the car flipping over

Lateral Weight Transfer

- Problem
  - 4 wheels deliver a statically overdetermined problem.
  - consider deformation to solve.
  - different components are suspended by different springs
    - Wheel, hub, … → tire
    - chassis → suspension + tire
    - influence of roll centre (RC)?
- Approach
  - We assume a linear system
  - Thus we can superimpose single causes
  - Split into
    - Non Suspended WT → tire, wheel, ½ suspension
    - Elastic WT → chassis mass rotating about RC
    - Geometric WT → chassis mass applied at RC



Lateral WT and CG's



(Claude Rouelle, Optimum G)

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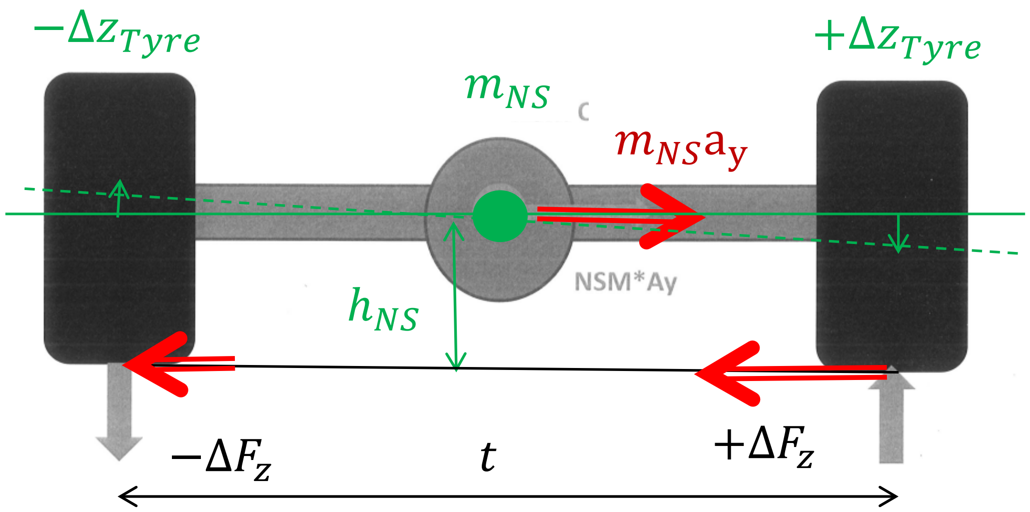
Non Suspended WT

- actually tyre suspended WT
- separated to front axle and rear axle
- the roll stiffness of an axle
- $$c_{roll} = \frac{M_x}{\varphi_x} = \frac{2\,\Delta F_z \frac{t}{2}}{\frac{\Delta z_{Tyre}}{\frac{t}{2}}} = \frac{t^2}{2} c_z$$
- High roll stiffness
  → quick WT
- No ARB-influence
- $$\Delta F_{z,NS} = \frac{M_{NS}}{t} = \frac{h_{NS}\, m_{NS}}{t} a_y$$

- e.g. Rigid Axle



WT of suspended mass and non suspended mass II

- Non Suspended WT

$$\Delta F_{z,NS} = \frac{h_{NS}}{t} m_{NS} a_y$$

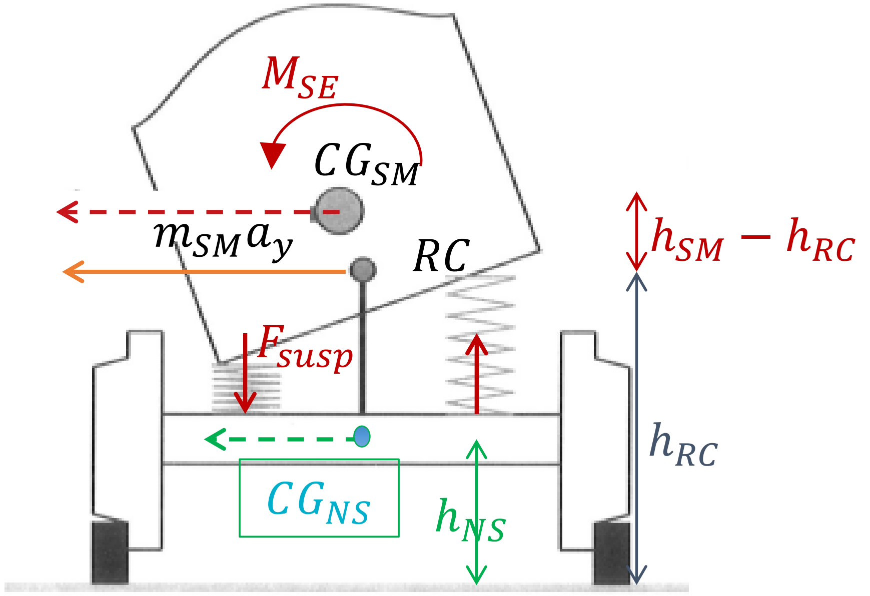
- „Geometric“ WT

$$\Delta F_{z,SG} = \frac{h_{RC}}{t} m_{SM} a_y$$

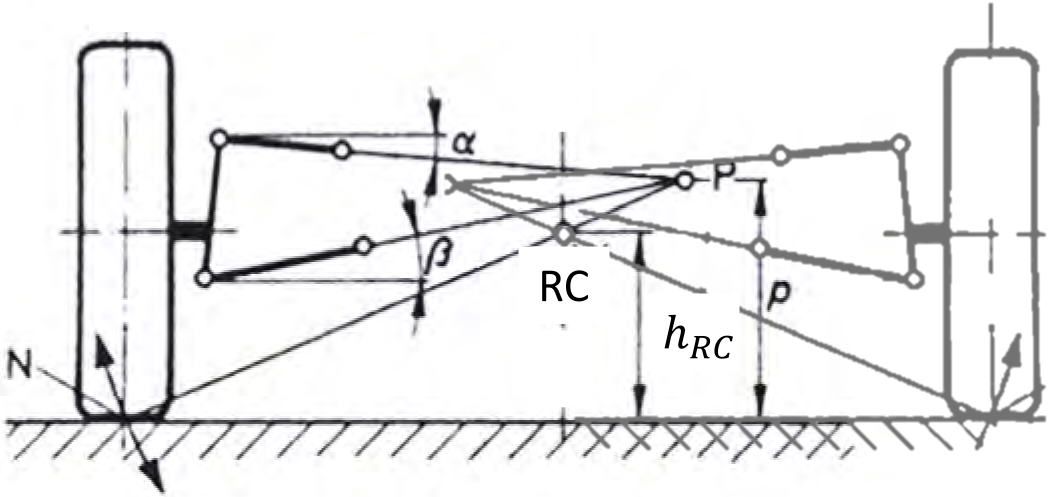
- „Elastic“ WT

$$\Delta F_{z,SE} = \frac{h_{SM} - h_{RC}}{t} m_{SM} a_y$$

- Non Suspended WT and Geometric WT acts **quickly**, Elastic WT acts slower, the suspension has to wind up.



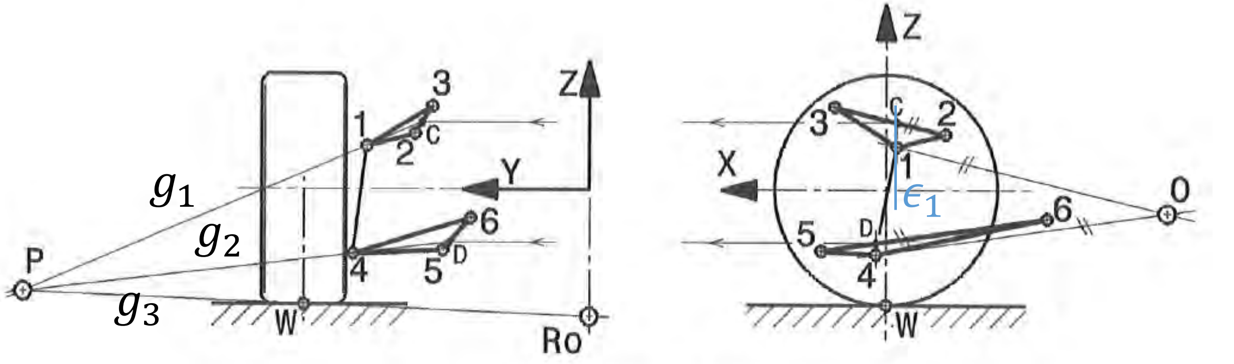
Wheel Suspension: Roll Centre

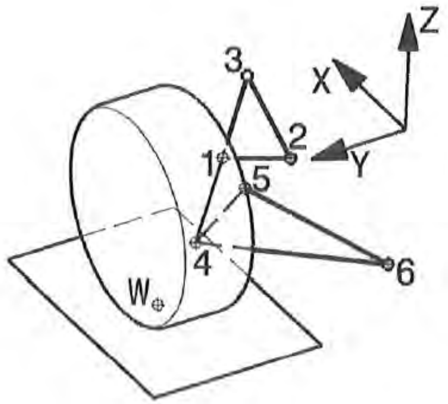


1. Connect instant centre of motion of the wheel relative to chassis with the contact point on left side
2. do same at right side
3. we get the Roll Centre at the intersection of the lines above (is not at y=0 in turns due to roll motion and asymmetry in the suspension!)

Roll Centre Ro of Double A-Arm with Anti-Feature

1. Point C: Intersect Axis $\overline{23}$ with a plane ϵ_1 parallel z-y-plane through 1
2. Project C and 1 to a z-y-parallel plane through wheel centre and connect these points with a line g_1.
3. Point D: Intersect Axis $\overline{56}$ with a plane parallel z-y-plane through 4
4. Project D and 4 to z-y-parallel plane through wheel centre and connect these points with a line g_2.
5. The intersection of g_1 and g_2 gives the instantaneous centre P for moving the wheel with fixed body.
6. Connect P to the centre of print W (= intersec. of wheel centre plane) to get g_3.
7. Intersect g_3 of left and right side to get Ro.





Konstruktion Rollzentrum Ro

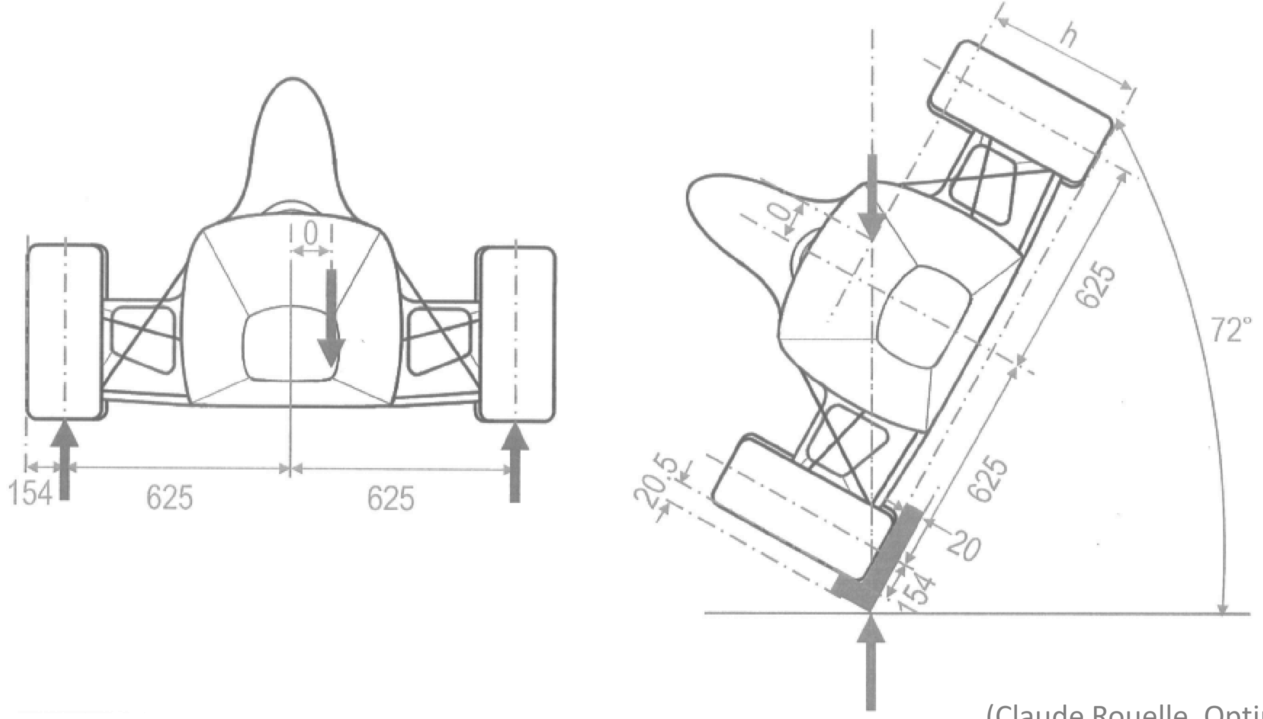
1. Seitenansicht: Punkte C und D = Schnittpunkte der Drehachsen $\overline{23}$ bzw. $\overline{56}$ mit der Senkrechten (Parallele zu Z-Achse) durch 1 bzw. 4.
2. Rückansicht: Querpol P = Schnittpunkt der Verbindungsgeraden $\overline{C1}$ und $\overline{D4}$.
3. Rückansicht: Rollzentrum Ro = Schnittpunkt der Verbindungsgeraden $\overline{PW}$ mit der Fahrzeugmittelebene.

Konstruktion Nickpol O

Seitenansicht: O = Schnittpunkt der Parallelen zu $\overline{23}$ durch 1 und der Parallelen zu $\overline{56}$ durch 4.

(Trzesniowski M.: Rennwagentechnik, Vieweg+Teubner 2008)

Find the exact CoG



(Claude Rouelle, Optimum G)

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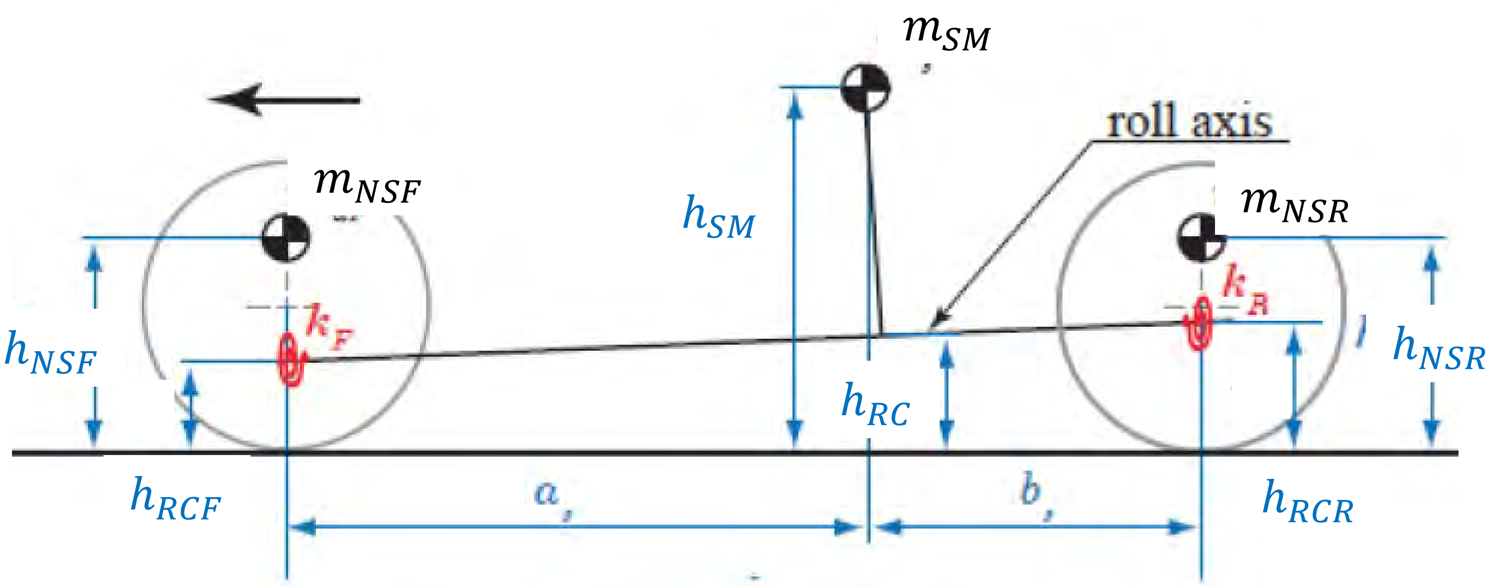
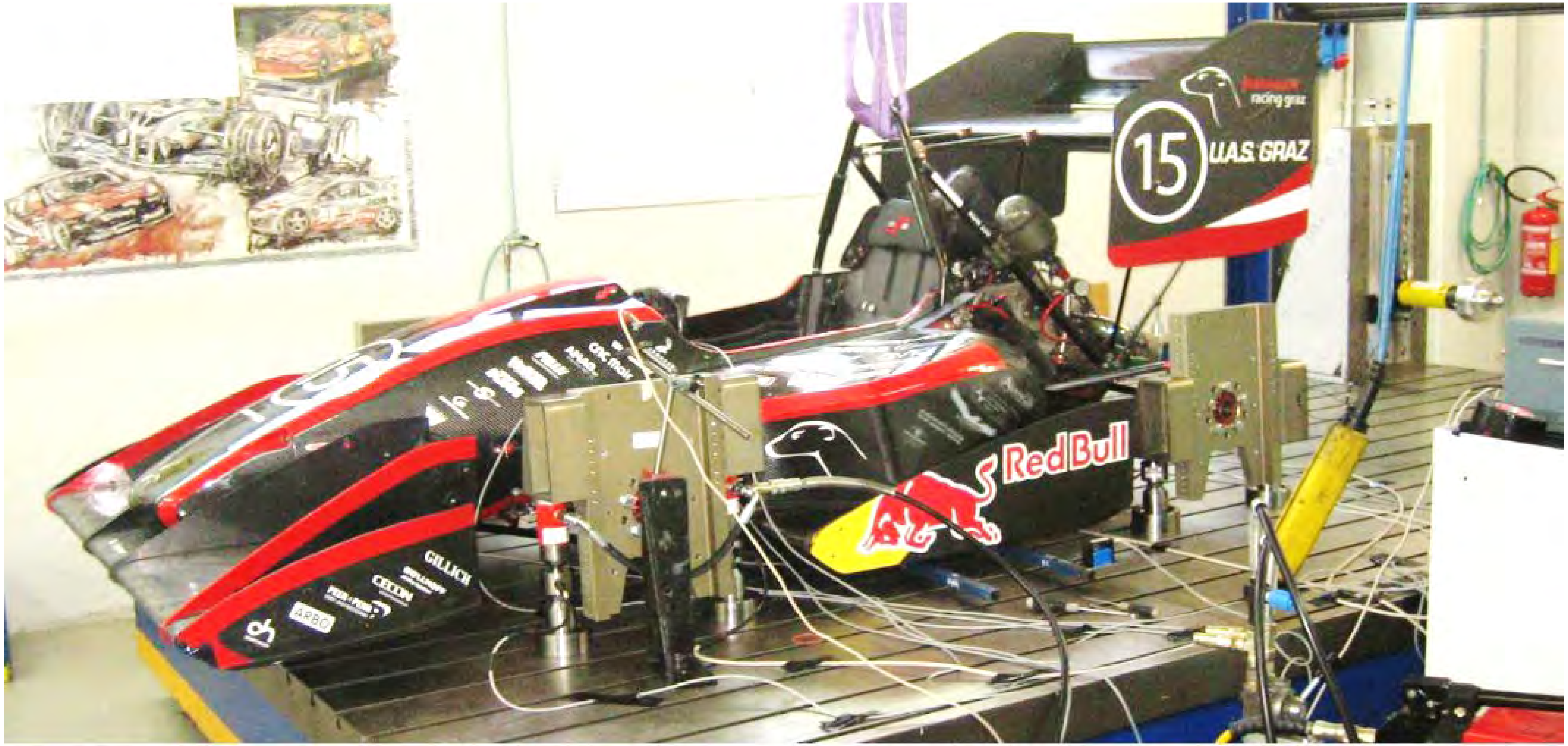
Elastic Weight Transfer

- chassis rotation axis:
  front RC to rear RC
- inertia force $F_{ySM} = m_{SM} \cdot a_y$ is applied in CG_{SM}
- has an arm $(h_{SM} - h_{RC})$
- the elastic part of roll moment $m_{SM}a_y(h_{SM} - h_{RC})$ is supported to front and rear according compliances.



Measurement of Chassis Compliance

Mount on rigid rim and block suspension!



(Kottnig G., Summer A.: Parameter of compliance, Seminar thesis AVD, 2015)

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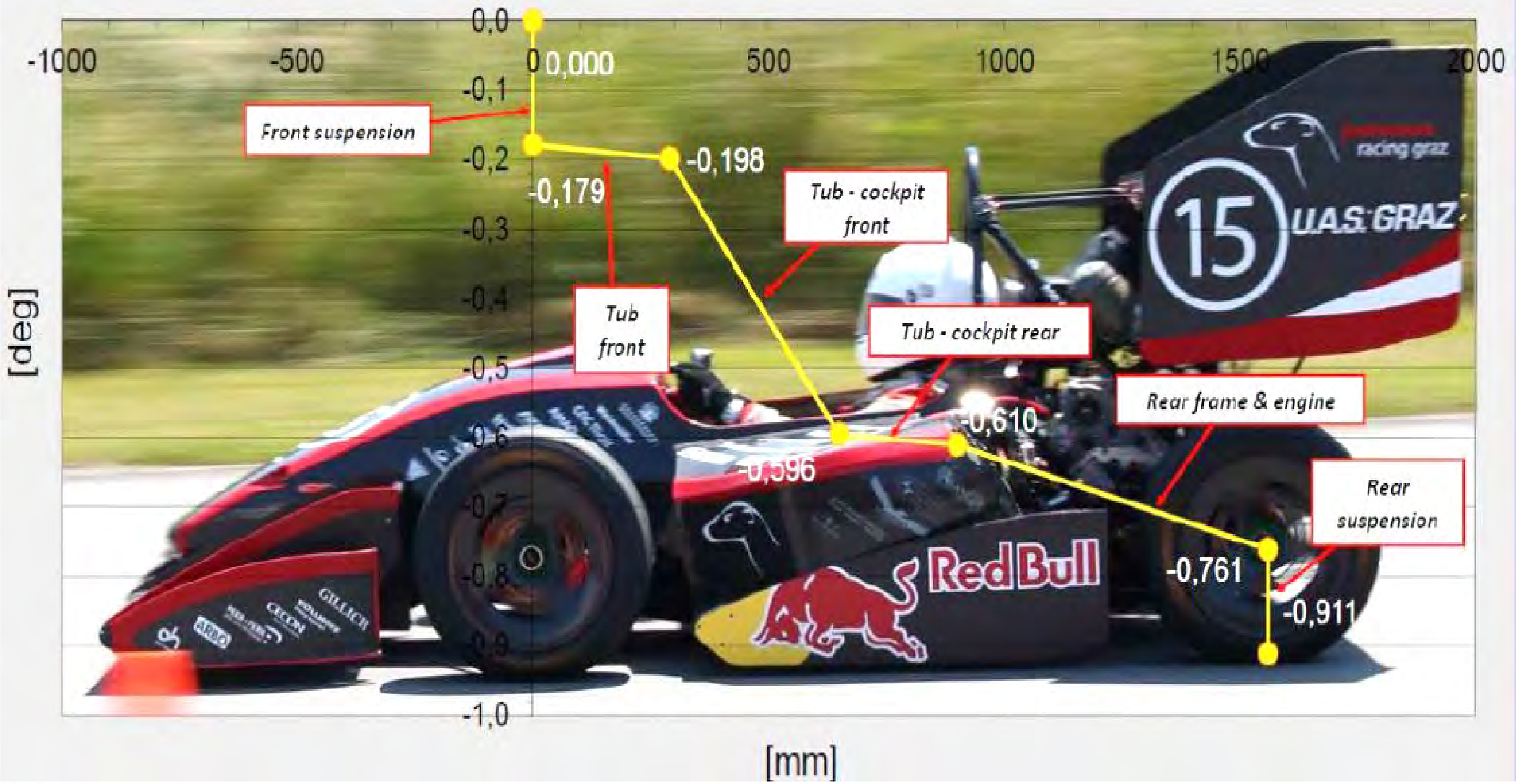
Chassis Compliance UAS FS15



(Kottnig G., Summer A.: Parameter of compliance, Seminar thesis AVD, 2015)

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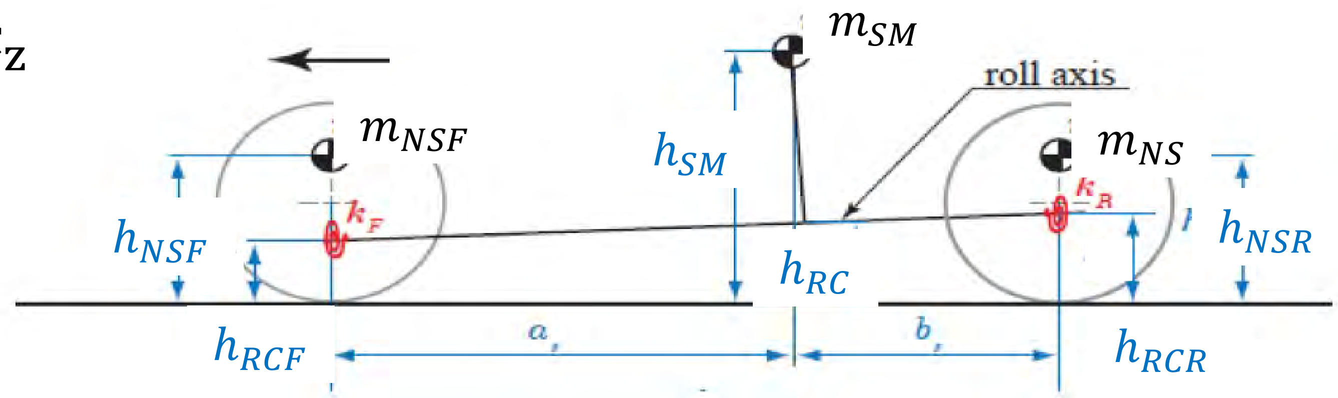
Torsional Springs in Series and Parallel

“elastic” roll moment to front

- Spring1F:
  Chassis compliance from CG to front
- Spring 2F: Roll stiffness of suspension + it’s compliance
- Spring 3F: front tyres $c_{roll} = \frac{t^2}{2} c_z$

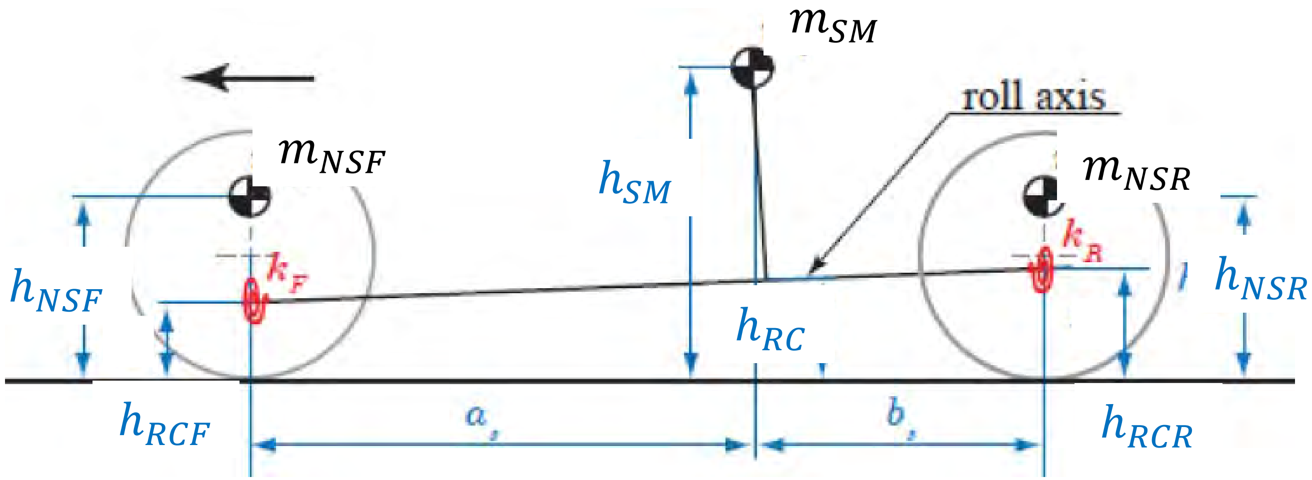
“elastic” roll moment to rear

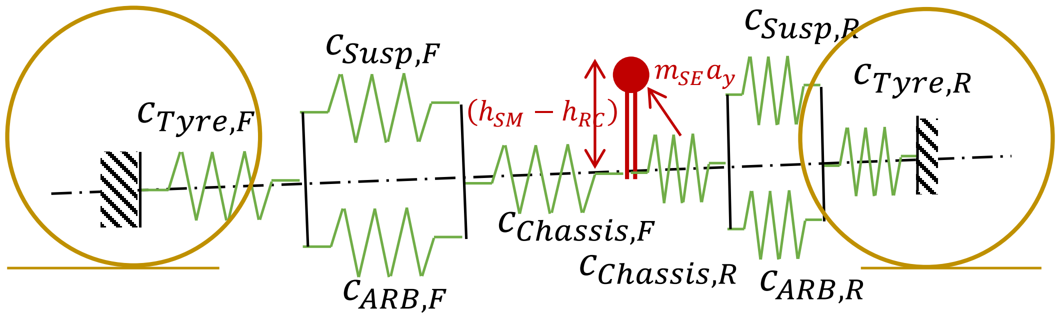
- Spring 1R: Chassis Compliance to rear
- Spring2R: total rear suspension
- Spring3R: rear tyres



Elastic Roll Moment Distribution

- Parallel connection of springs
- sum of stiffness
- $$c_{parallel} = \sum_i c_i$$
- Series connection of springs
- sum up the compliances
- $$\frac{1}{c_{series}} = \sum_i \frac{1}{c_i}$$





Geometric Roll Moment Distribution

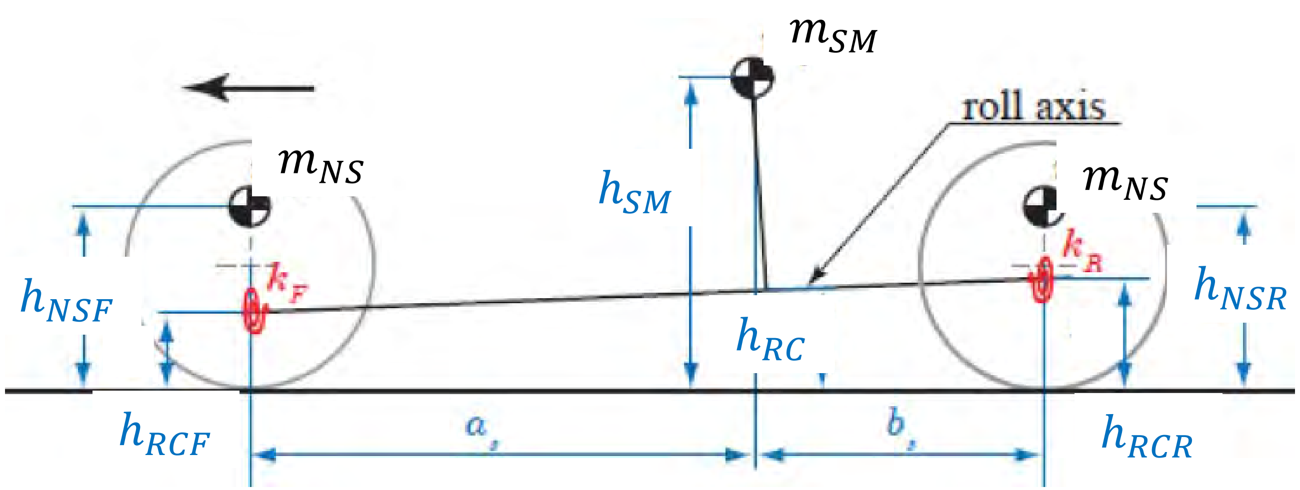
- The geometric part produces no Moment around the roll axis

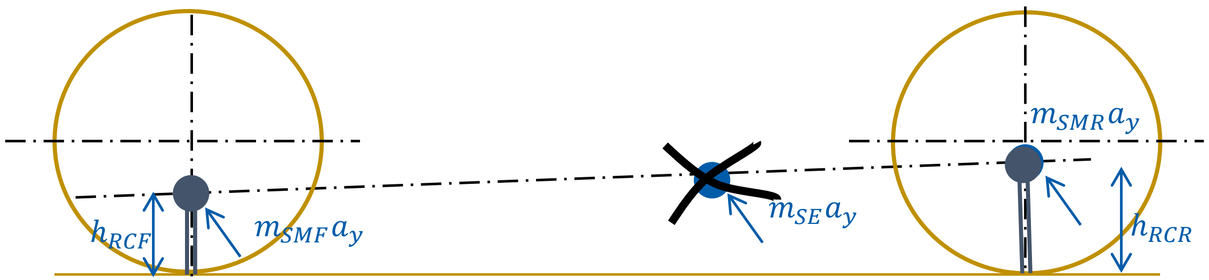
→ lever rule delivers a split into front and rear

$$m_{SMF} = \frac{b_S}{a_S + b_S} m_{SM}$$

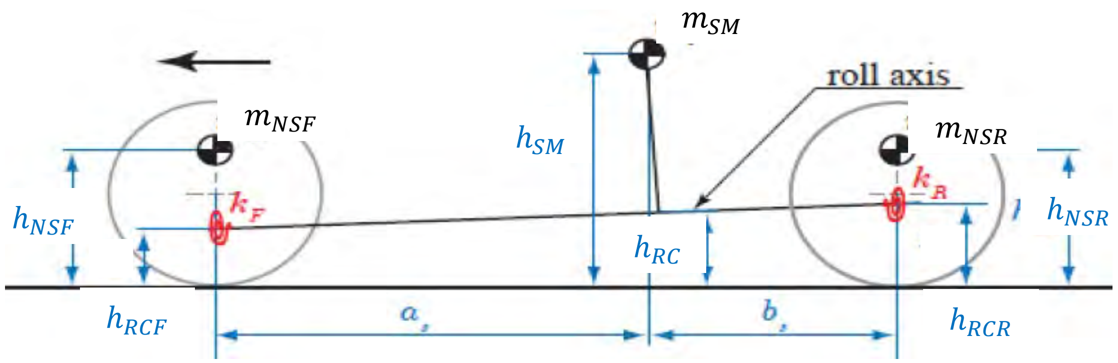
$$m_{SMR} = \frac{a_S}{a_S + b_S} m_{SM}$$

- handle like NSM





2nd Homework: Weight transfer



Wanted:

- Determine the weight transfer ΔF_{zay} for $a_y = 1\frac{m}{s^2}$ of inner and outer front wheels.
- Determine the weight transfer ΔF_{zax} for $a_x = 1\frac{m}{s^2}$ acceleration of front and rear wheels.

- The suspended mass $m_{SM} = 220\ kg$. It's CG is $a = 1.2\ m, b = 0.8m,\ h_{SM} = 0.25\ m,$.
- Front and rear suspensions are similar. The mass of 2 wheels, wheel hubs, half of suspensions is $m_{NS} = 30$ kg. It's CG is located at $h_{NS} = 0.25$ m.
- The roll axis is determined by $h_{RCF} = 0.05\ m, h_{RCR} = 0.1\ m$. Front and rear track width is $s = 1.5\ m$.
- The compliance for roll motion is described by the stiffness' of front suspension $c_{suspF} = 3000\ \frac{Nm}{rad}$, rear suspension $c_{suspR} = 3500\ \frac{Nm}{rad}$,chassis to front $c_{chassisF} = 80000\ \frac{Nm}{rad}$, chassis to rear $c_{chassisR} = 160000\ \frac{Nm}{rad}$. There is one front anti roll bar with a stiffness of $c_{ARBF} = 20000\ \frac{Nm}{rad}$ and no rear stabilizer bar. The tyre stiffness $c_{tz} = 100\frac{kN}{m}$.

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Solution

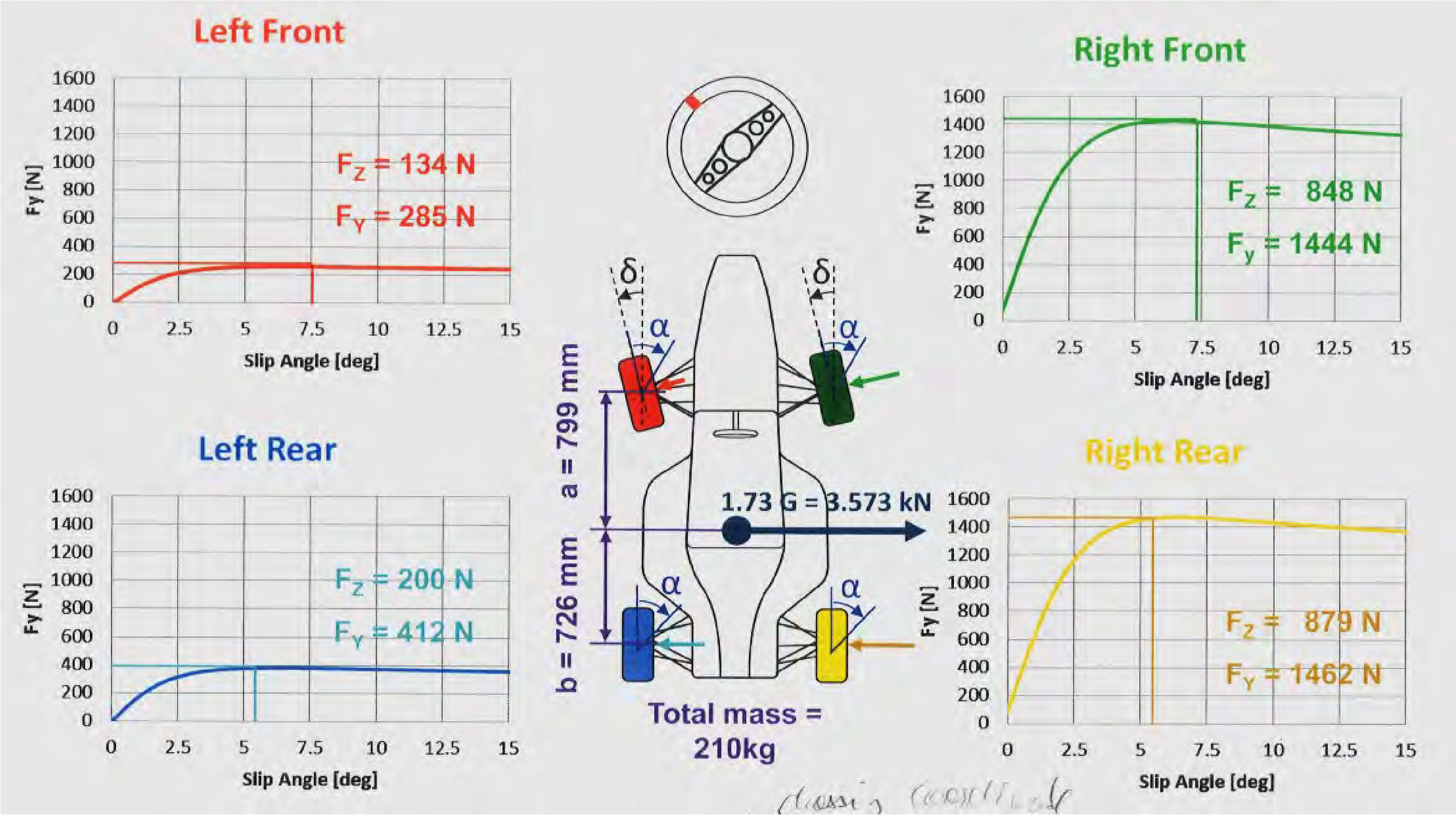
Full 2-Track-Model

for wide radii

- $R \gg l_{WB}$:
- $\alpha_{FL} \cong \alpha_{FR}$
- $\alpha_{RL} \cong \alpha_{RR}$

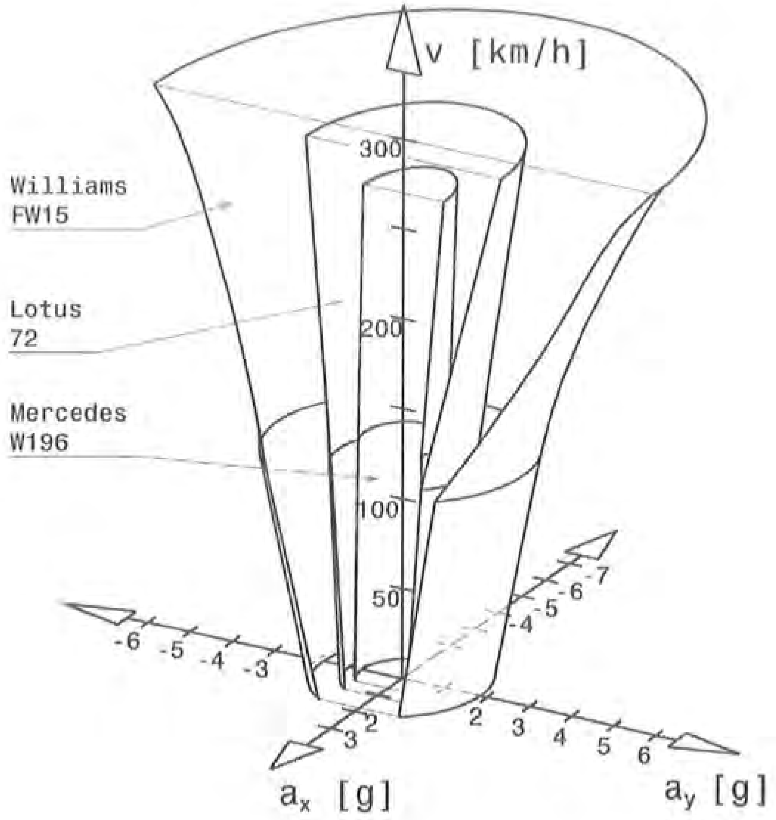
else consider

- kinematics
- steering gear ratio



(Claude Rouelle, Optimum G)

g-g-diagram Formula-1



(Trzesniowski: Rennwagentechnik 2008)

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2Track Model vs. Single Wheel Model

UNITED

2-Track Model

- WT is considered
- higher accuracy
- g-g- and yaw-moment-diagram
- usable to optimize components
  - tyre‘s wheel load influence
  - camber
  - mass distribution
  - compliances, ARB‘s, suspension springs
  - suspension kinematics
  - ...
- a lot of parameters must be known to set up

Single Wheel Model

- No WT, no tyre load dependence
- lower accuracy
- only g-g-diagram
- usable to optimize path
- delivers an idealized vehicle
- good results to find accelerations limits

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Simplified Model to obtain the g-g-diagram

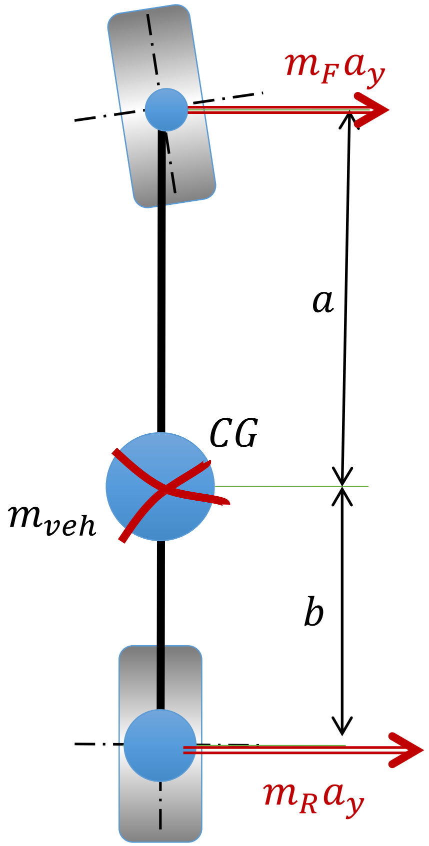
- Mono-Cycle with wheel-load from bicycle model.
  - split into front and rear axle mass
    $m_F = \frac{m_{veh} b}{a+b}, m_R = \frac{m_{veh} a}{a+b}$
  - Axle load front: $F_{zF} = \frac{1}{2}(m_F g + F_{lift,F})$
  - Lateral Force: $F_{yF} = \frac{1}{2}(m_F a_y)$

    ... assuming steady state => $M_{yaw} = 0$
  - Longitudinal Force:

$$F_{xF} = \frac{1}{2}(m_{veh} a_x + F_{Drag}) \cdot k_{AWDF}$$

$$k_{AWDF} = \frac{M_{DriveF}}{M_{DriveF} + M_{DriveR}}$$

  - Inclination Angle $\gamma = 0$, steering angle $\delta \ll 1$
  - Same with rear axle, one axle reaches the limit earlier



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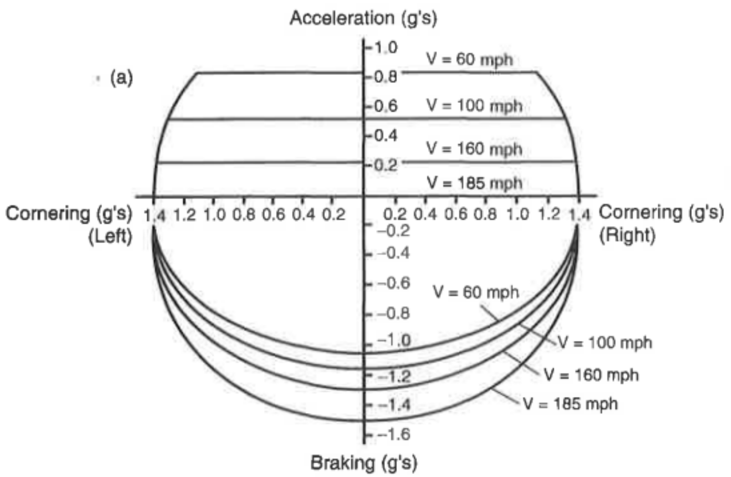
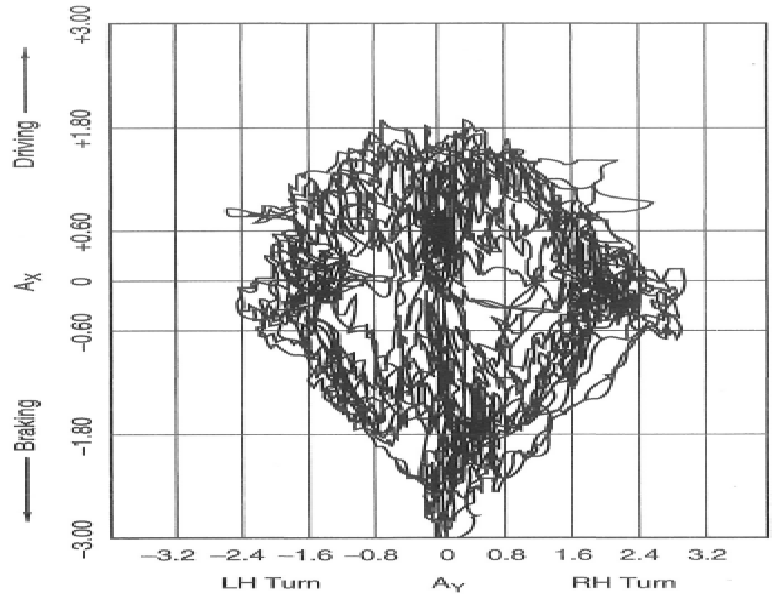
g-g-diagram: $a_x(a_x, v_x)$

theoretical=potential



measured=driver's courage



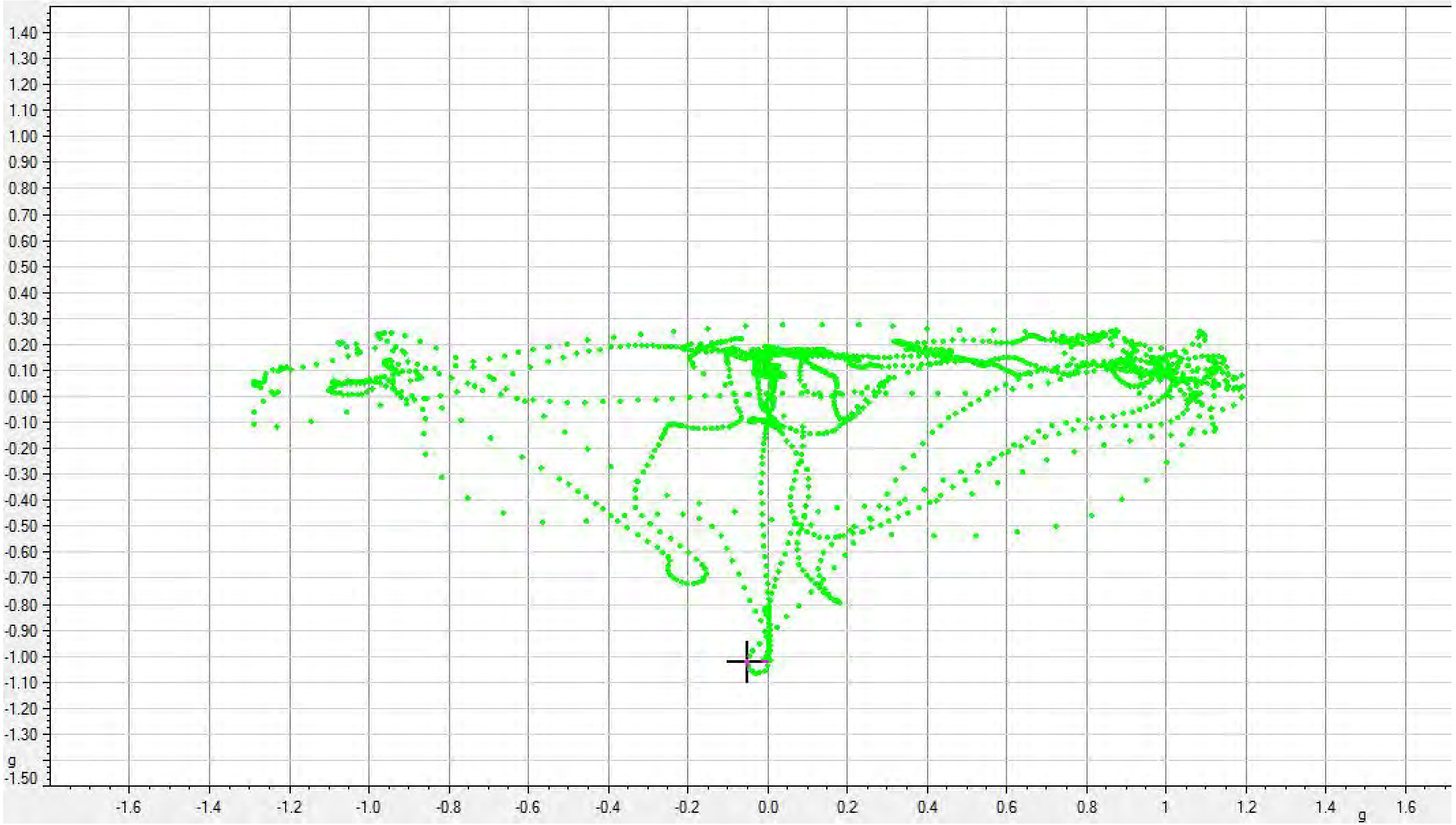
Practice on dry circuit.
(Reference personal communication
Peter Wright, Team Lotus.)

Figure 9.7 Adelaide, 1987, Senna.

[Milliken W.,Milliken D.: Race Car Vehicle Dynamics, SAE 1995]

Measured g-g-diagram

- How far does the driver use the limits?



passenger car
(http://www.trailbrake.net/featured-articles/the-g-g-diagram)

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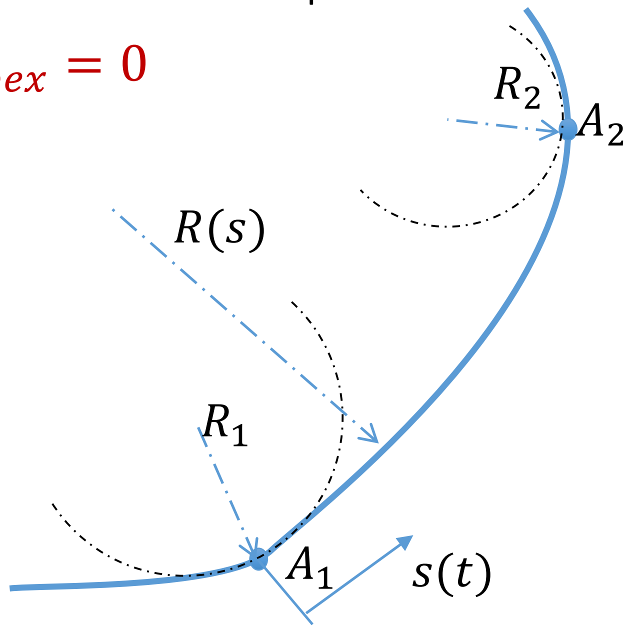
How can we pass the lap as fast as possible?

- G-G-diagram
- strait forward
  - radius $R = \infty$, curvature $\kappa = \frac{1}{R} = 0$
  - No lateral force F_y
  - Full longitudinal force F_x to accelerate/decelerate is possible
- The shape of a turn is defined by it's curvature/radius along the path length $s(t)$
  - the radius $R(s)$ decreases continuously until reaching the apex point (Scheitelpunkt)

- No longitudinal force in apex point delivers maximal speed
  - Brake before apex
  - Tire potential in apex is used fully for lateral acceleration.
  - Accelerate after apex
- $F_{x,Apex} = 0$



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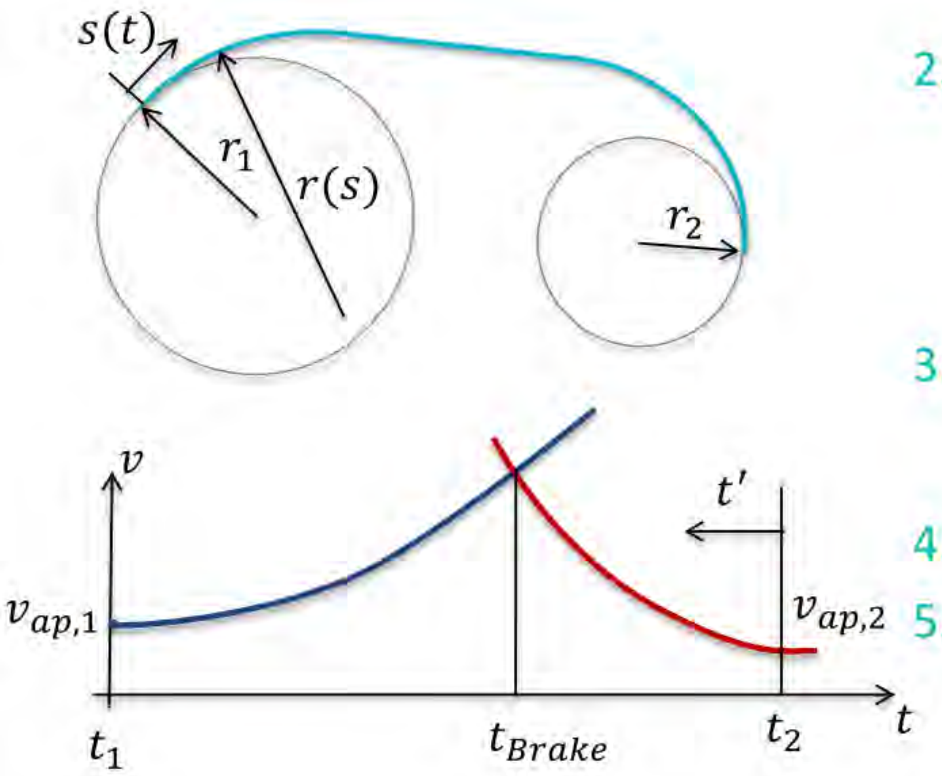
Lap Time Simulation with known path



1. *Speed in Apex*

   $a_x = 0, a_y = \frac{v_{ap1,2}^2}{r_{1,2}} \rightarrow v_{ap\ 1,2} = \sqrt{a_{y,max} r_{1,2}}$

2. Calc. Acceleration

   $v(t) = \int_{t1}^{t} a_x(s) dt^*$, IC: $v_1 = v_{ap,1}$

   $a_x(t) = f(a_y, v(t))$... g-g-diagram, $a_y = \frac{v(t)^2}{r(s)}$

   $s = s(t) = \int_{t1}^{t} v(t) dt^*$

3. Calc. Decelleration (offline)

   analog topic 2) but starting at Apex2 using negative time t'

4. The intersection is the braking point

5. Alternatively

   Calculate decelleration recursively and check the speed at apex 2.

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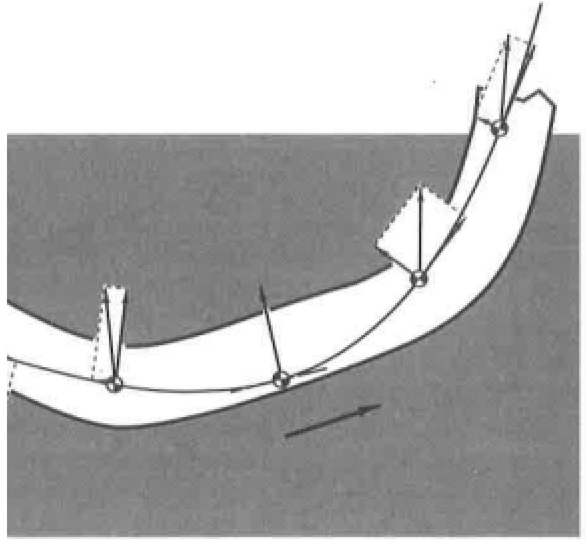
Trip Time Simulation 2

2. accelerate after apex A_1

   - current lateral force due to curvature and speed
     - $F_y(\kappa, v)$
   - tire's potential in x
     - $F_{x,Ty}\left(F_y, F_z(v)\right)$, e.g $\left(\frac{F_x}{F_x^M}\right)^2 + \left(\frac{F_y}{F_y^M}\right)^2 = 1$
   - engine's potential
     - e.g. $F_{x,Eng} = \frac{P_{Eng} \cdot \eta}{v_u}$
   - current drag
     - $F_{Drag}(v, s)$
   - long. acceleration
     - $F_{acc} = m_{tot} \cdot a_x = F_x - F_{Drag}$
   - integration of $a_x(t)$ to get speed $v(t)$ and path length $s(t)$

- We solve a 1st Order ODE starting at apex point.



[Milliken W.,Milliken D.: Race Car Vehicle Dynamics, SAE 1995]

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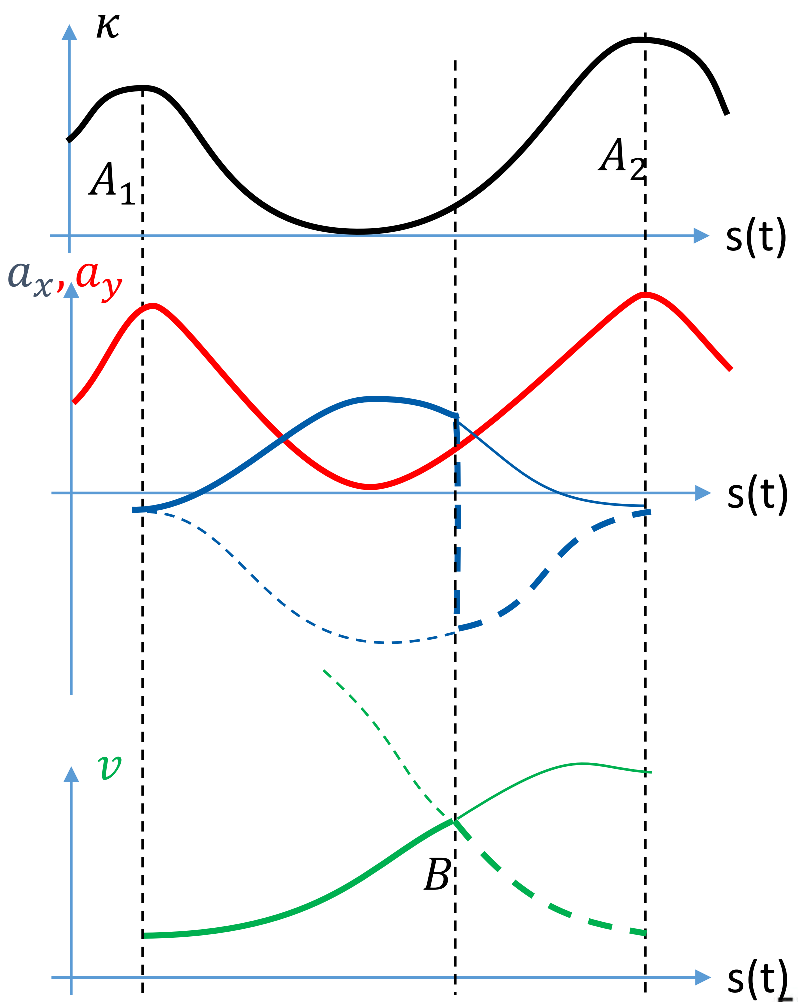
Trip Time Simulation 3

3. brake to next apex A_2
   a) find brake point B recursively
      - try a brake point
      - brake as fast as possible due to tire's potential
      - check speed in next apex and correct brake point
   b) calc. speed out of next apex using negative time
      - accelerate in reverse direction with brake potential starting at next apex.
      - brake point is the intersection of the $v(s(t))$ curves

- We brake in that way, that we reach the maximum possible speed in next apex.
- Single point model delivers a goal, which can be reached by optimal car setup



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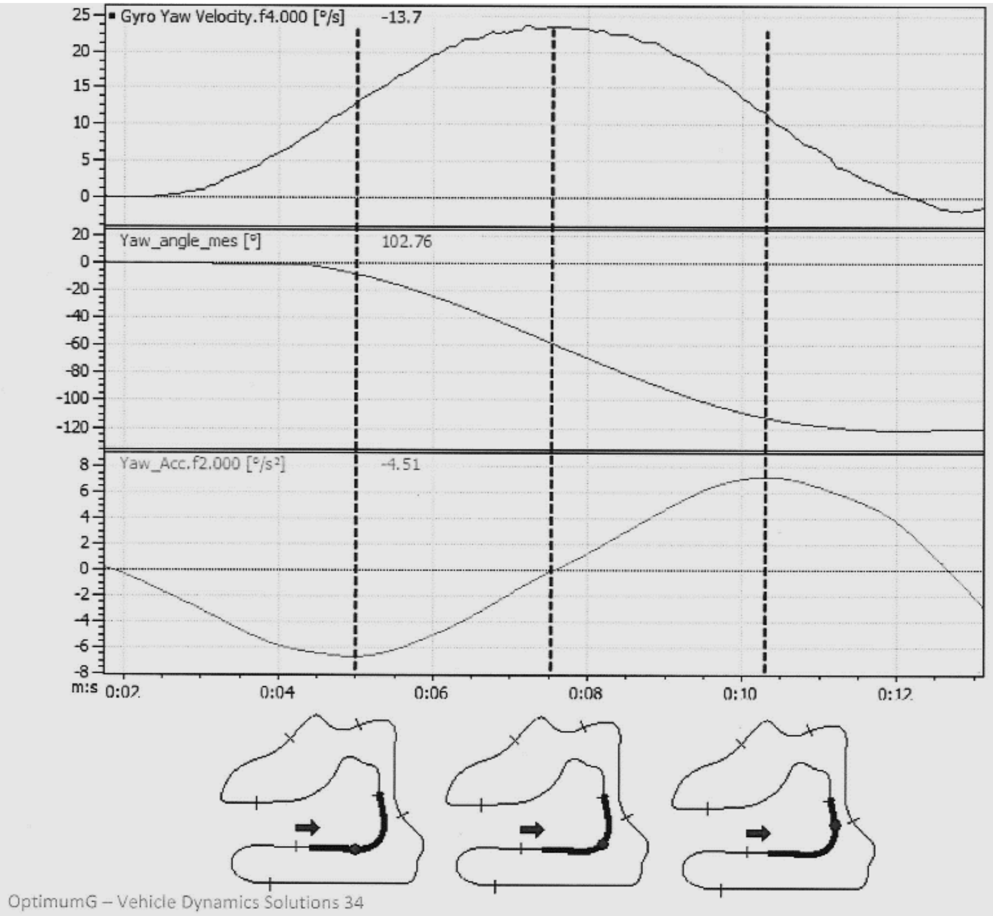
Turning Manoeuvre



(Claude Rouelle, Optimum G)

- Change Velocity Direction
  - accelerate the body laterally

$$a_y = \frac{v^2}{R}$$

- Change Heading Angle ψ
  - yaw acceleration $\ddot{\psi} > 0$ before apex point
  - yaw deceleration $\ddot{\psi} > 0$ after apex point

- Newton‘s Law

$$m_{veh} \cdot a_y = \sum F_{y,i}$$
$$I_{zz} \cdot \ddot{\Psi} = \sum M_{z,i}$$

We need force **and torque** to make the turn!

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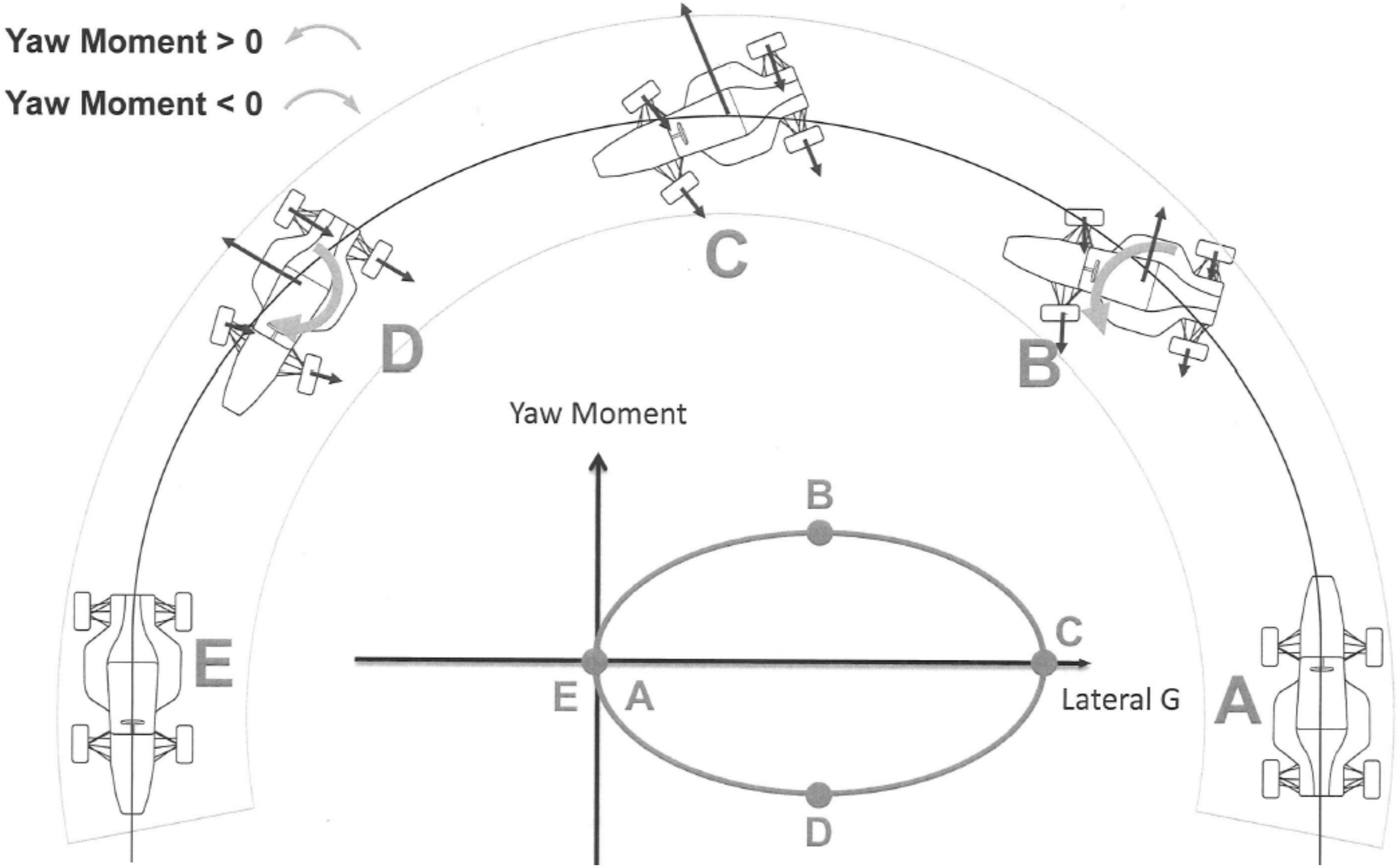
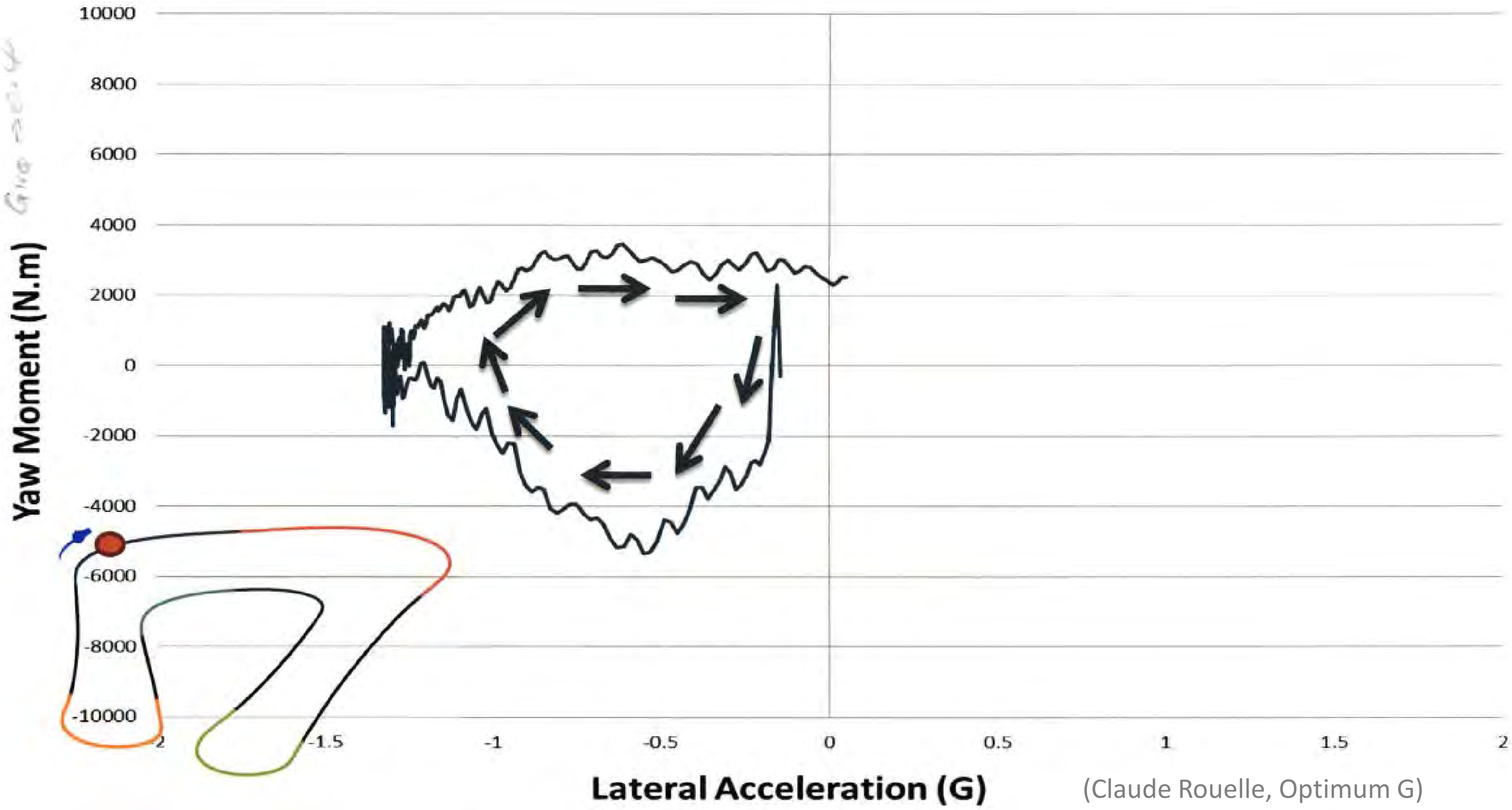
Yaw Moment and Lateral Acceleration



(Claude Rouelle, Optimum G)

Yaw Moment vs. Lateral Accelleration



(Claude Rouelle, Optimum G)

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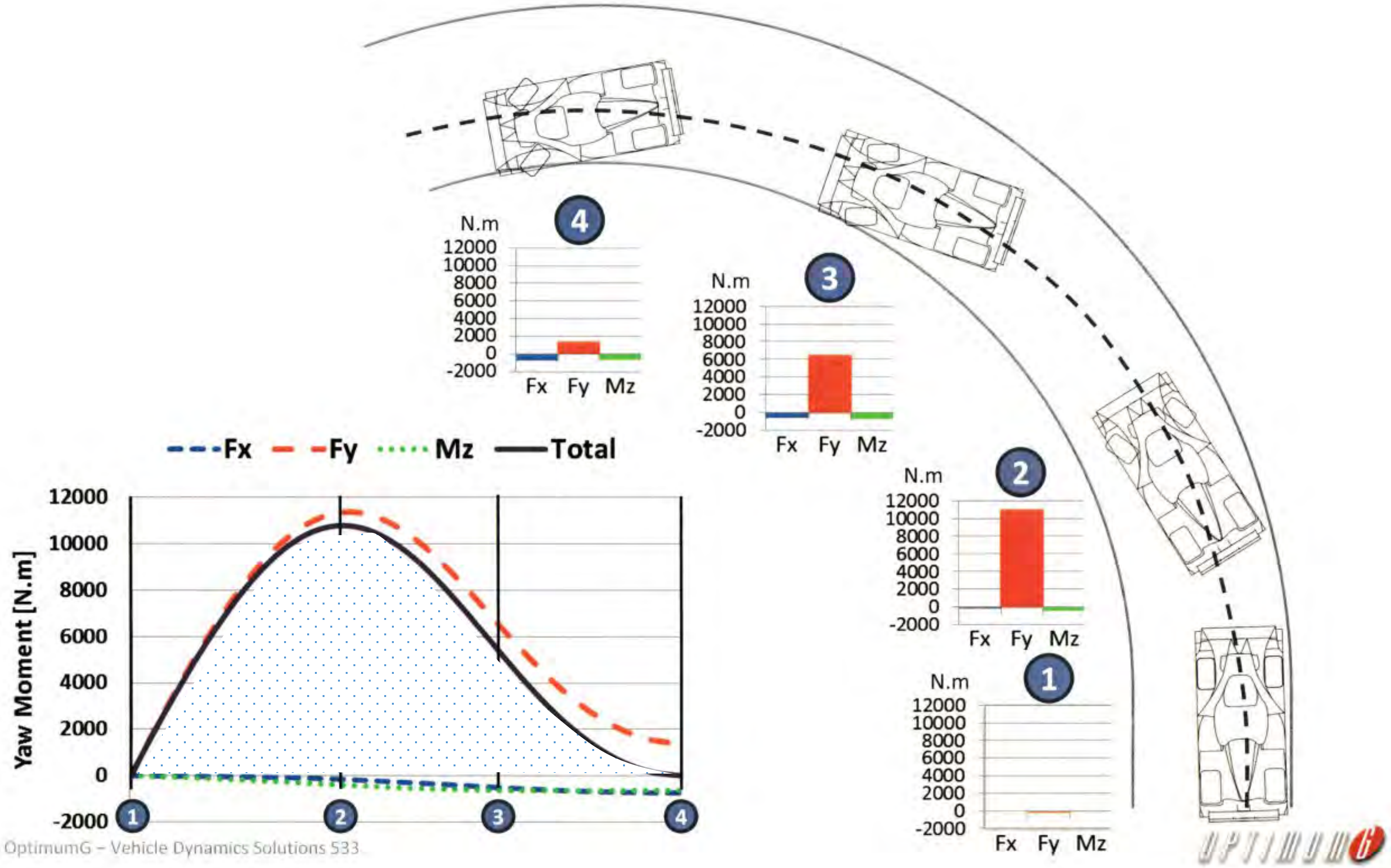
Yaw Moment while Entering a Corner

$$I_{zz}\frac{d\dot{\Psi}}{dt} = M_z$$

$$E_{kin,rot} = I_{zz} \cdot \frac{\dot{\Psi}^2}{2} = \int M_z \, dt$$



(Claude Rouelle: OptimumG)

Tyre model delivers forces in tyre coordinate system

- Tyre Model delivers for
  - longitudinal slip,
  - lateral slip
- Tyre Forces $F_x . F_y$
  - applied in tyre CS
- Torques
  - self alignment torque M_z due to trail and deformation
  - Overturn torque M_x due to deformation and inclination angle
- Drive Torque
  - $M_y = r_e \cdot F_y$

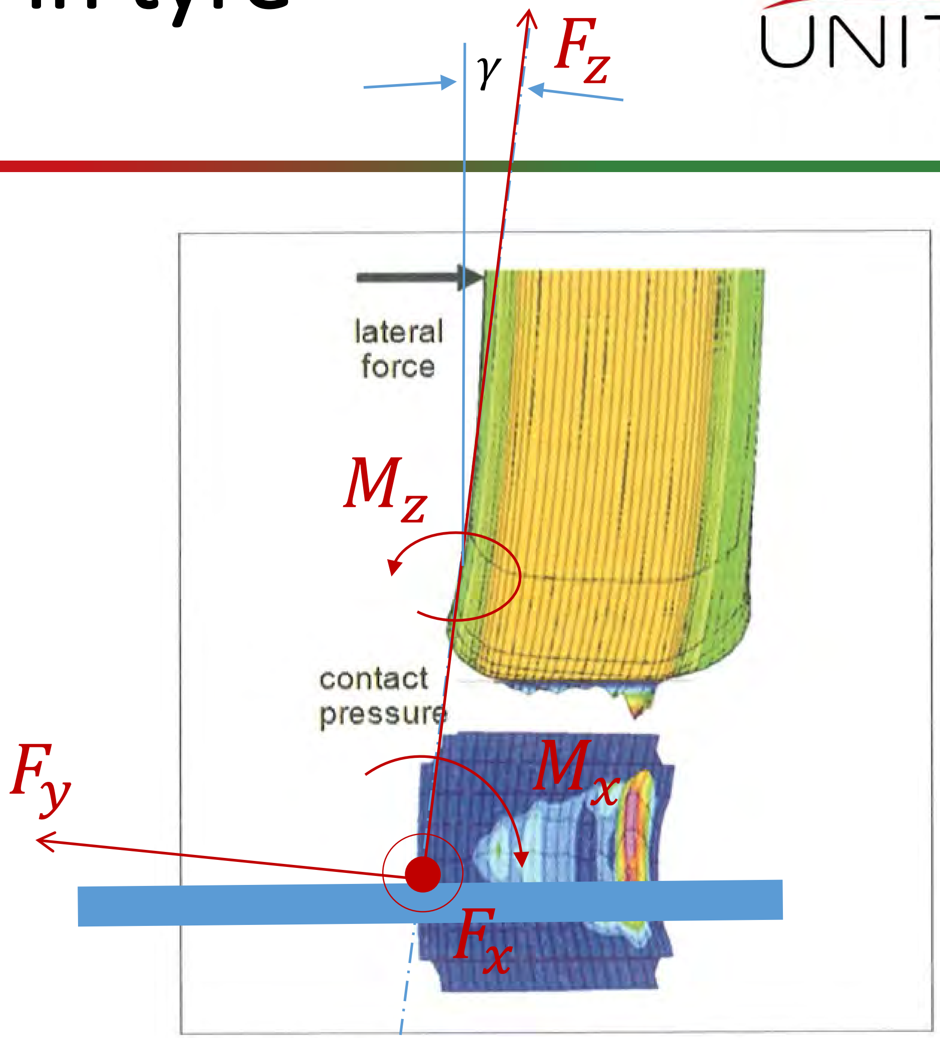
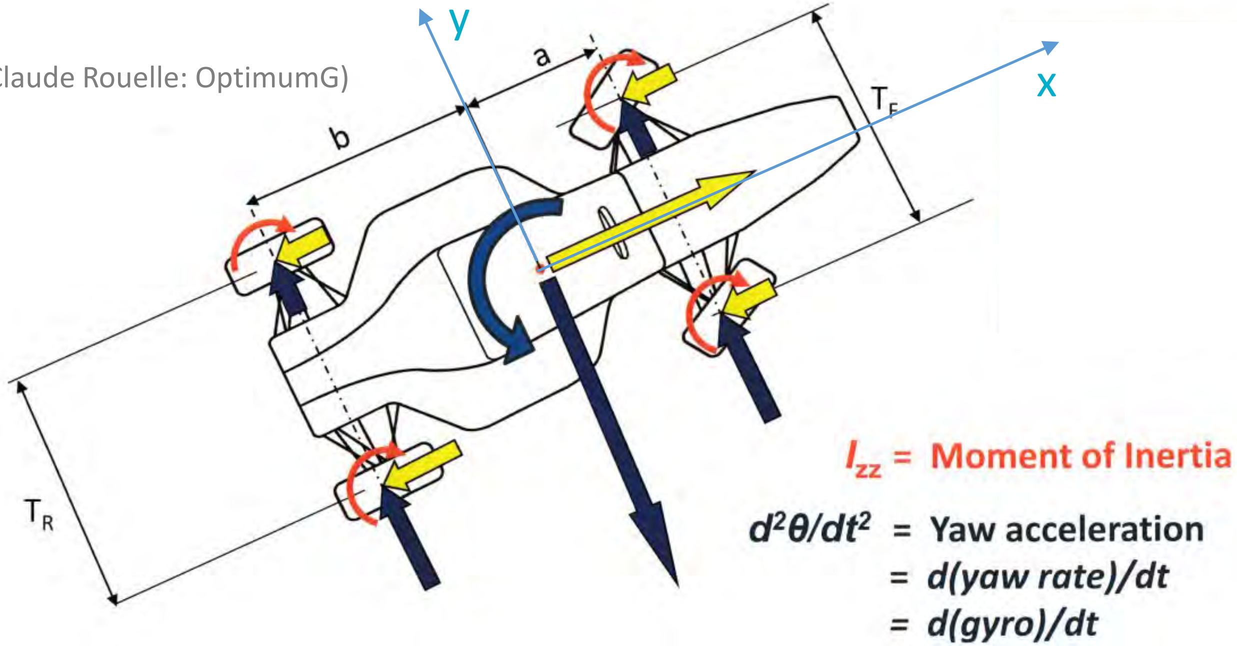


Fig. 3-433: FEM model and calculated contact pressure for a steady-state rolling tire with sideslip

Forces applied on vehicle body

(Claude Rouelle: OptimumG)



Turn

$$m_{veh} \cdot a_y = F_{lat}$$
$$I_{zz} \cdot \ddot{\Psi} = M_{yaw}$$

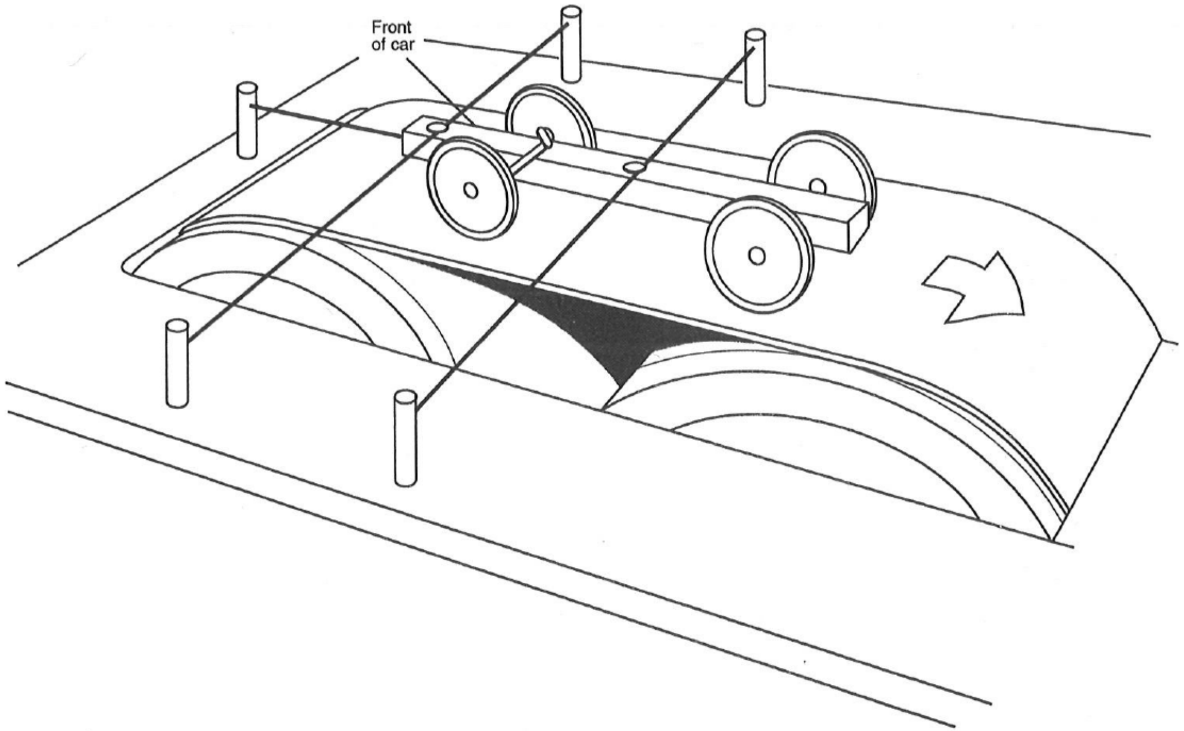
Milliken-Moments-Diagram:

- Rolling $m_{veh} \cdot a_x = 0$
- for given $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$ we get side slip angles and tyre forces
- resultant forces F_{lat}, M_{yaw}
- diagram $M_{yaw}\ vs.\ a_y$

$$F_{lat} = (F_{yFL}cos\delta_L + F_{yFR}cos\delta_R) + (F_{yRL} + F_{yRR}) + [+\mathrm{F}_{\mathrm{xFL}}\sin(\delta_L) + \mathrm{F}_{\mathrm{xFR}}\sin(\delta_R)$$
$$M_{yaw} = (F_{yFL}cos\delta_L + F_{yFR}cos\delta_R)a - (F_{yRL} + F_{yRR})b + \sum M_{z,i} +$$
$$[-\mathrm{F}_{\mathrm{xFL}}\sin(\delta_L) + \mathrm{F}_{\mathrm{xFR}}\sin(\delta_R) - \mathrm{F}_{\mathrm{xRL}} + F_{xRR}]$$

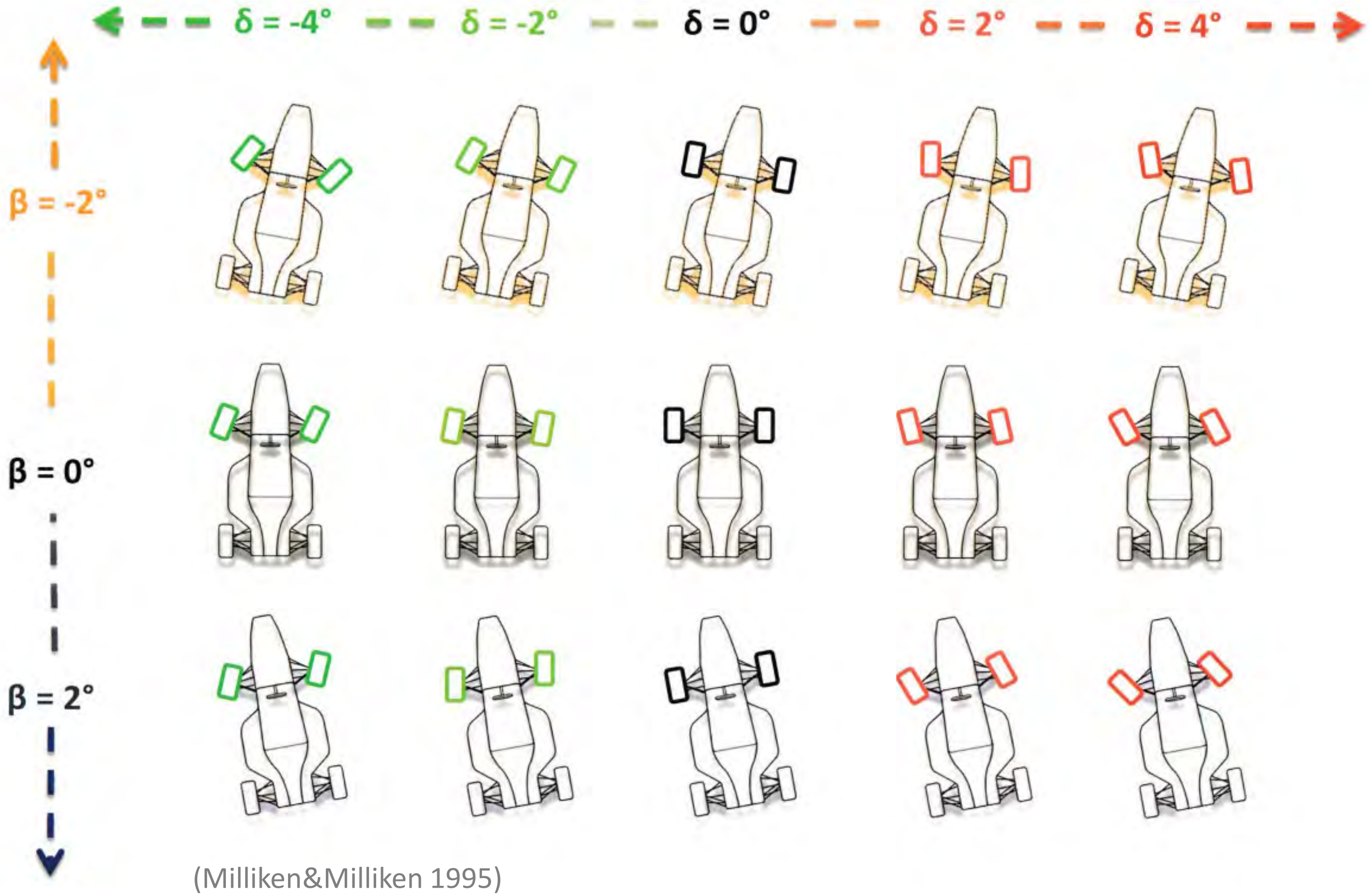
Milliken Moments Diagram
= Yaw Moment vs. ay Diagram

We change steering angle δ and body slip angle β and measure F_y and M_z.



(Milliken&Milliken 1995)

Lateral Force and Yaw Moment vs. β, δ



(Milliken&Milliken 1995)

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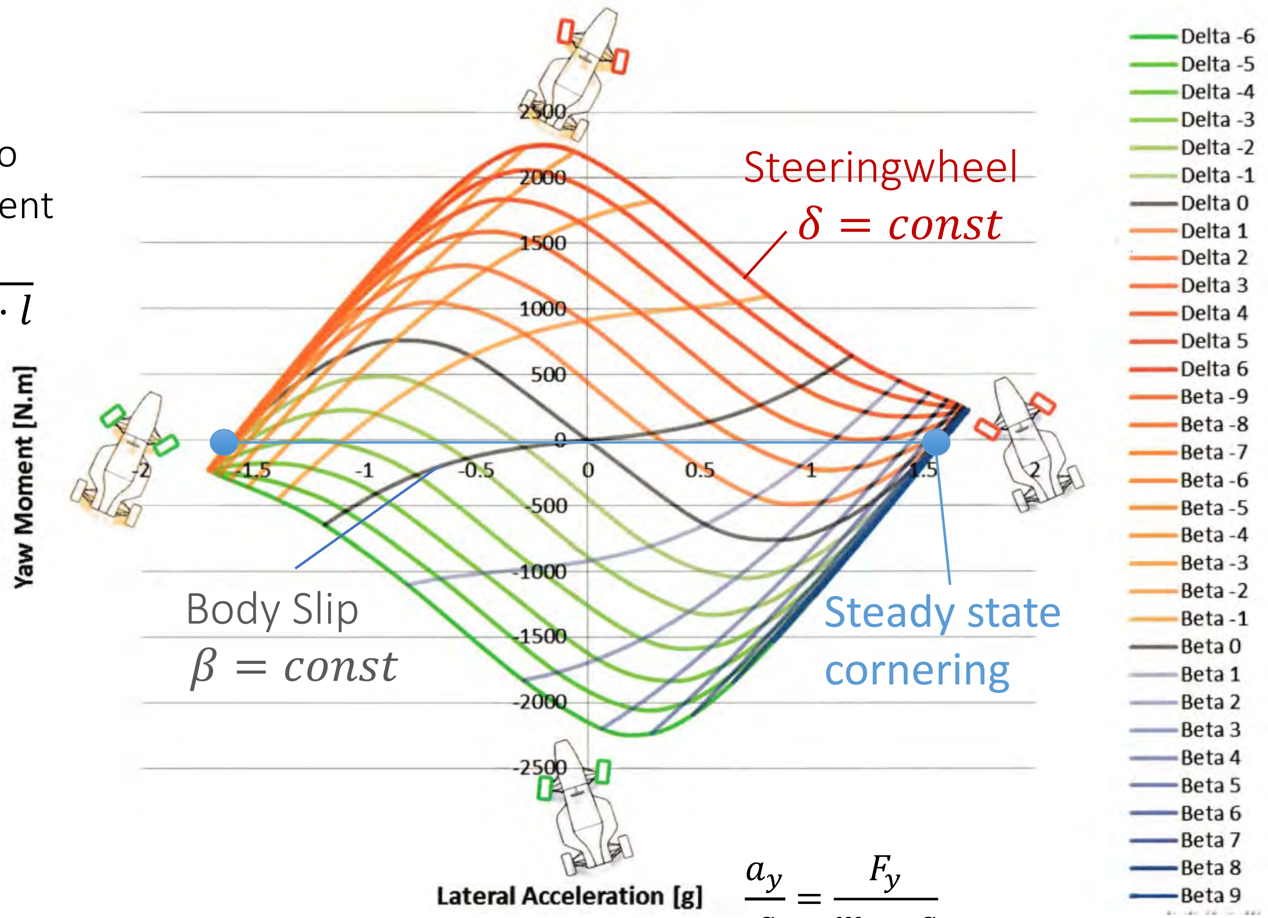
YMD

or we can also scale the moment

$$C_n = \frac{M_z}{m_{veh} g \cdot l}$$



$$\frac{a_y}{g} = \frac{F_y}{m_{veh} g}$$

SAE-Coordinate are used here!
(Claude Rouelle: OptimumG)

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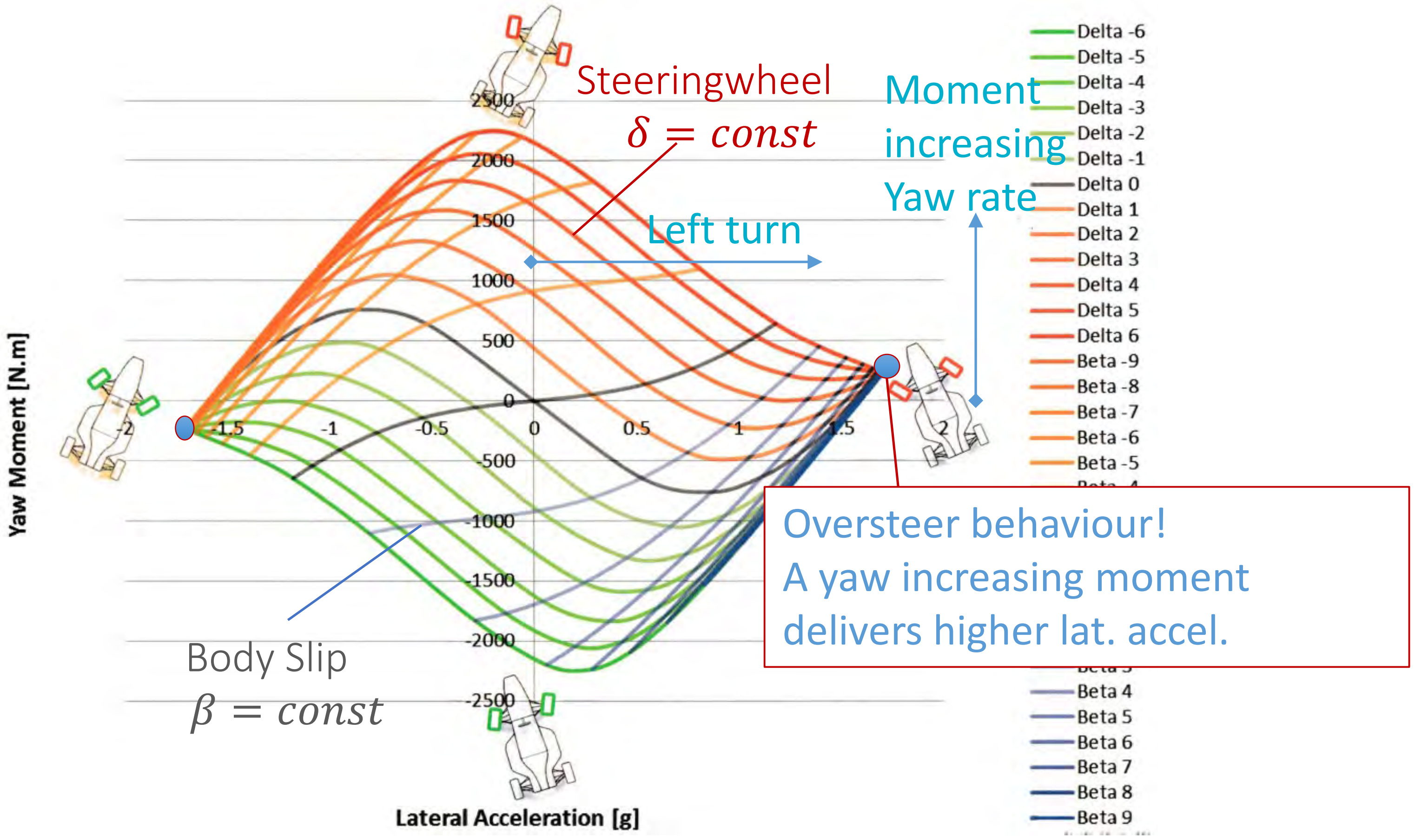
Force-Moment-Analysis, Yaw-Moment Analysis, Milliken Moments Diagram

- … is a Steady State Force Analysis, of the unbalanced car, we neglect
  - tyre dynamics - Tyre needs about print length L to build up forces
  - damper‘s influence in load transfer
- Gives answers to
  - Controllability $\frac{dM_z}{d\delta}$>0
    - Does an increase of δ cause an tighter turn?

    →An increase of δ increases Yaw Moment (=yaw acceleration for more yaw angle)
  - Stability $\frac{dM_z}{d\beta}$<0
    - Is there a backing torque, $M_z < 0$, if β drifts from equilibrium?

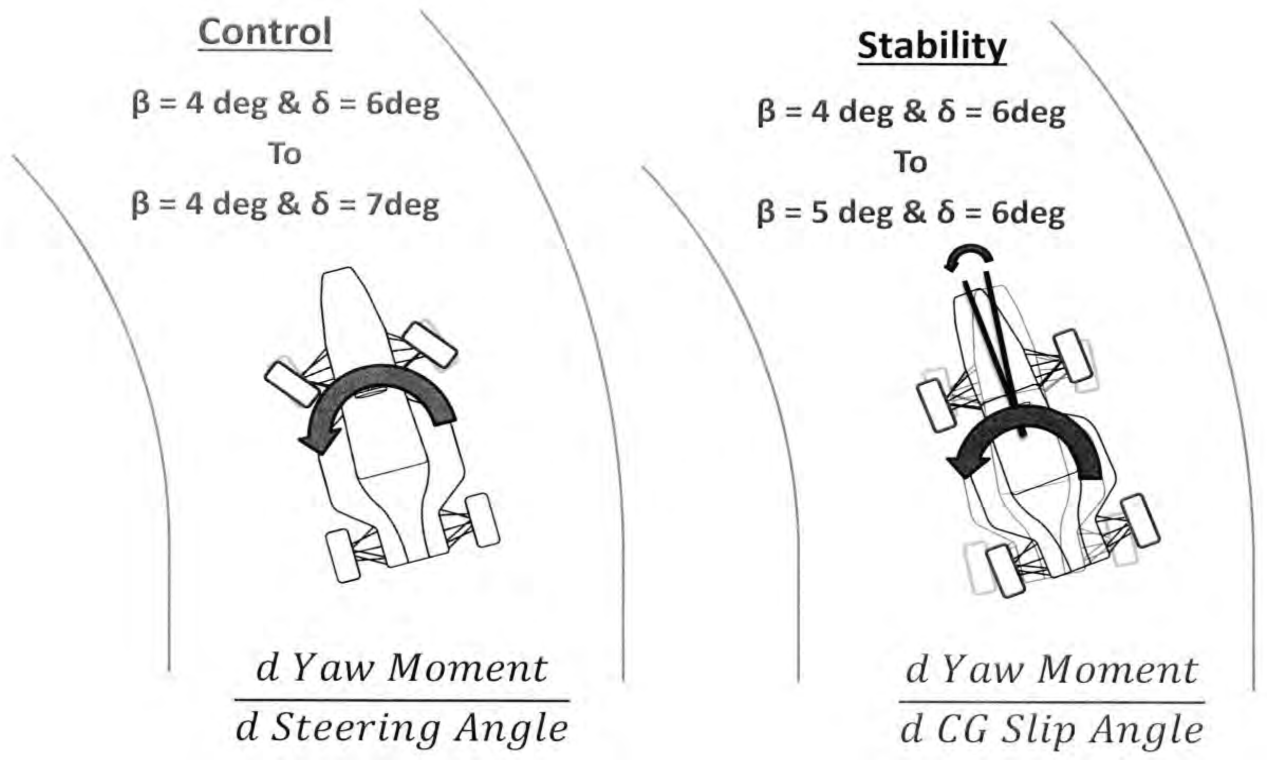
    → A increase of β decreases Yaw Moment (=yaw acceleration backwards reducing β)

YMD – Understeer/oversteer



SAE-Coordinate are used!
(Claude Rouelle: OptimumG)

Stability & Controllability: Inclination in YMD



Does an increase of δ increase yaw rate?

Does an increase of β produce a moment moving back?

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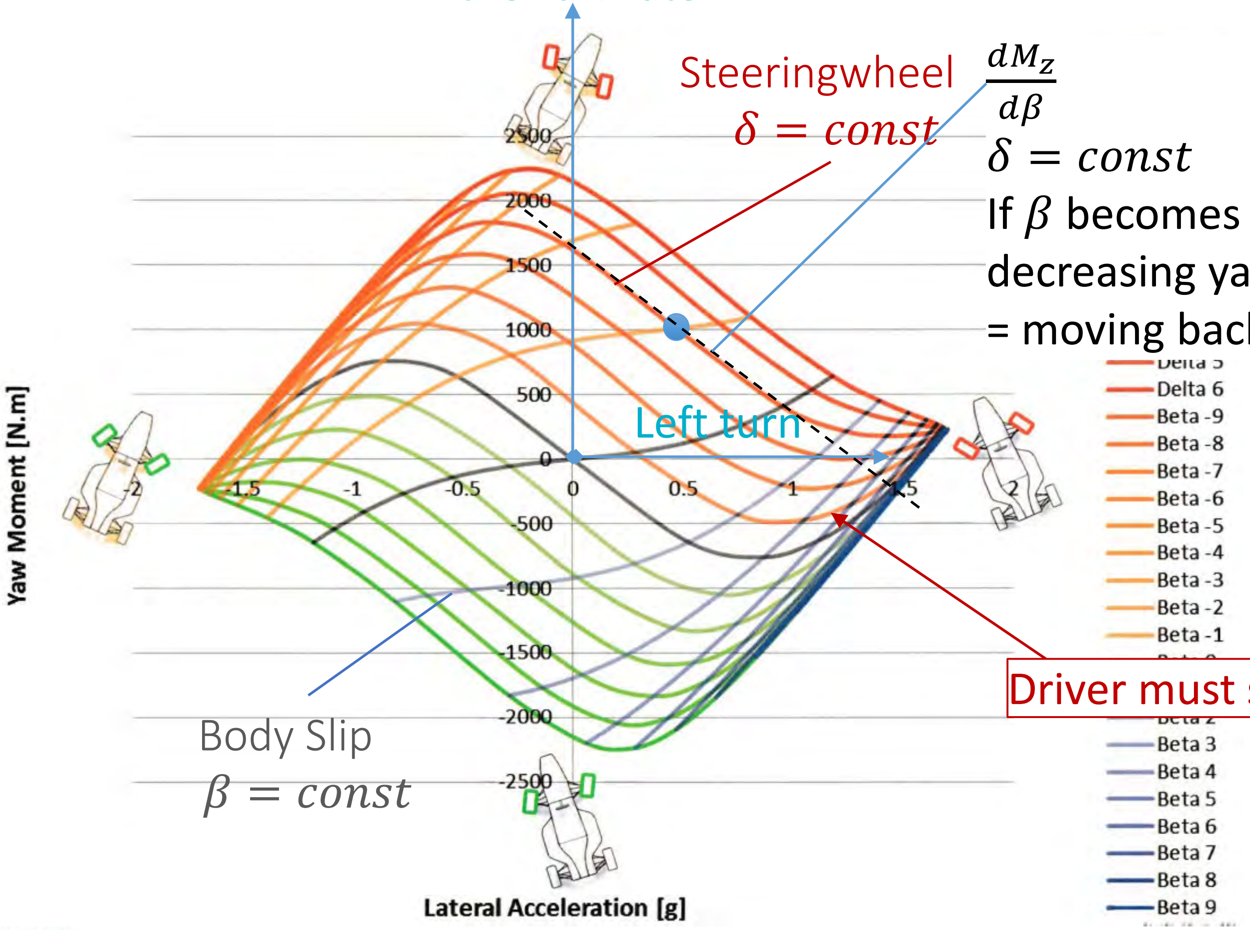
YMD – Stability



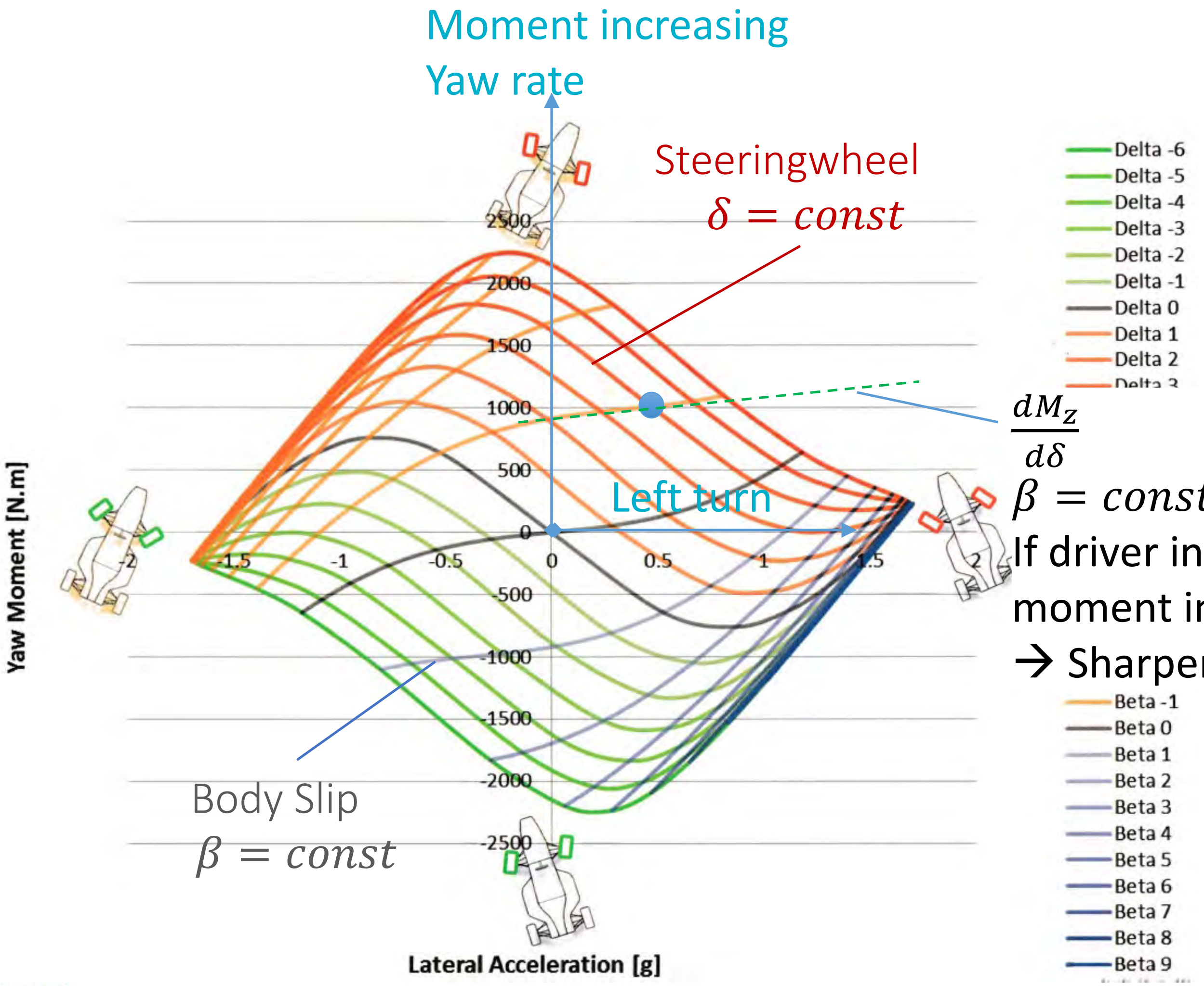
$\delta = const$

If β becomes larger we have a moment decreasing yaw rate

= moving back → stable

SAE-Coordinate are used!
(Claude Rouelle: OptimumG)

YMD – Control(ability)



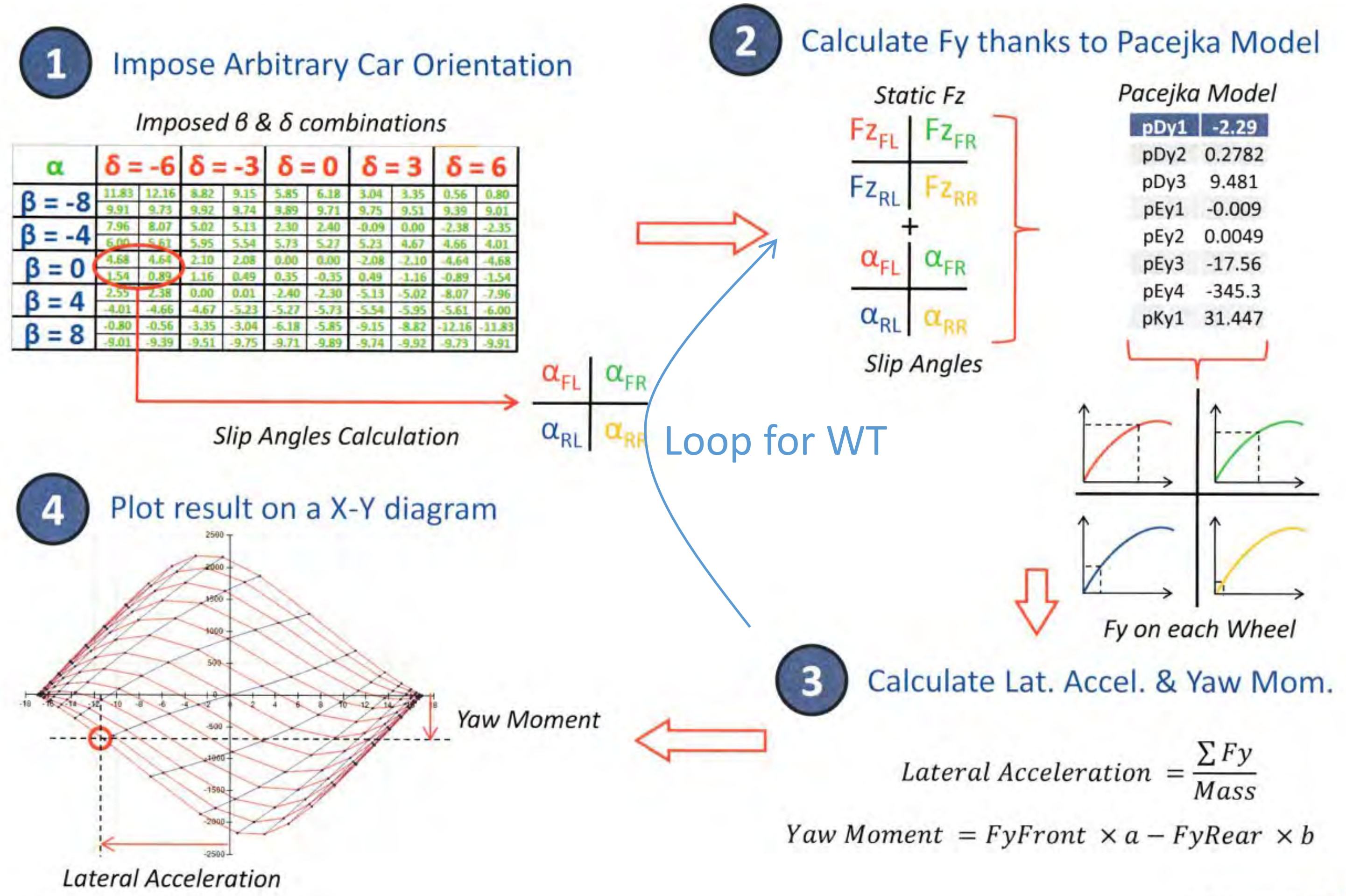
$\beta = const$

If driver increases δ we get an moment increasing yaw rate.

→ Sharper turn

SAE-Coordinate are used!
(Claude Rouelle: OptimumG)

YMD Flowchart



α	δ = -6		δ = -3		δ = 0		δ = 3		δ = 6	
β = -8	11.83	12.16	8.82	9.15	5.85	6.18	3.04	3.35	0.56	0.80
	9.91	9.73	9.92	9.74	9.89	9.71	9.75	9.51	9.39	9.01
β = -4	7.96	8.07	5.02	5.13	2.30	2.40	-0.09	0.00	-2.38	-2.35
	6.00	5.61	5.95	5.54	5.73	5.27	5.23	4.67	4.66	4.01
β = 0	4.68	4.64	2.10	2.08	0.00	0.00	-2.08	-2.10	-4.64	-4.68
	1.54	0.89	1.16	0.49	0.35	-0.35	0.49	-1.16	-0.89	-1.54
β = 4	2.55	2.38	0.00	0.01	-2.40	-2.30	-5.13	-5.02	-8.07	-7.96
	-4.01	-4.66	-4.67	-5.23	-5.27	-5.73	-5.54	-5.95	-5.61	-6.00
β = 8	-0.80	-0.56	-3.35	-3.04	-6.18	-5.85	-9.15	-8.82	-12.16	-11.83
	-9.01	-9.39	-9.51	-9.75	-9.71	-9.89	-9.74	-9.92	-9.73	-9.91

$$Lateral\ Acceleration = \frac{\sum Fy}{Mass}$$

$$Yaw\ Moment = FyFront \times a - FyRear \times b$$

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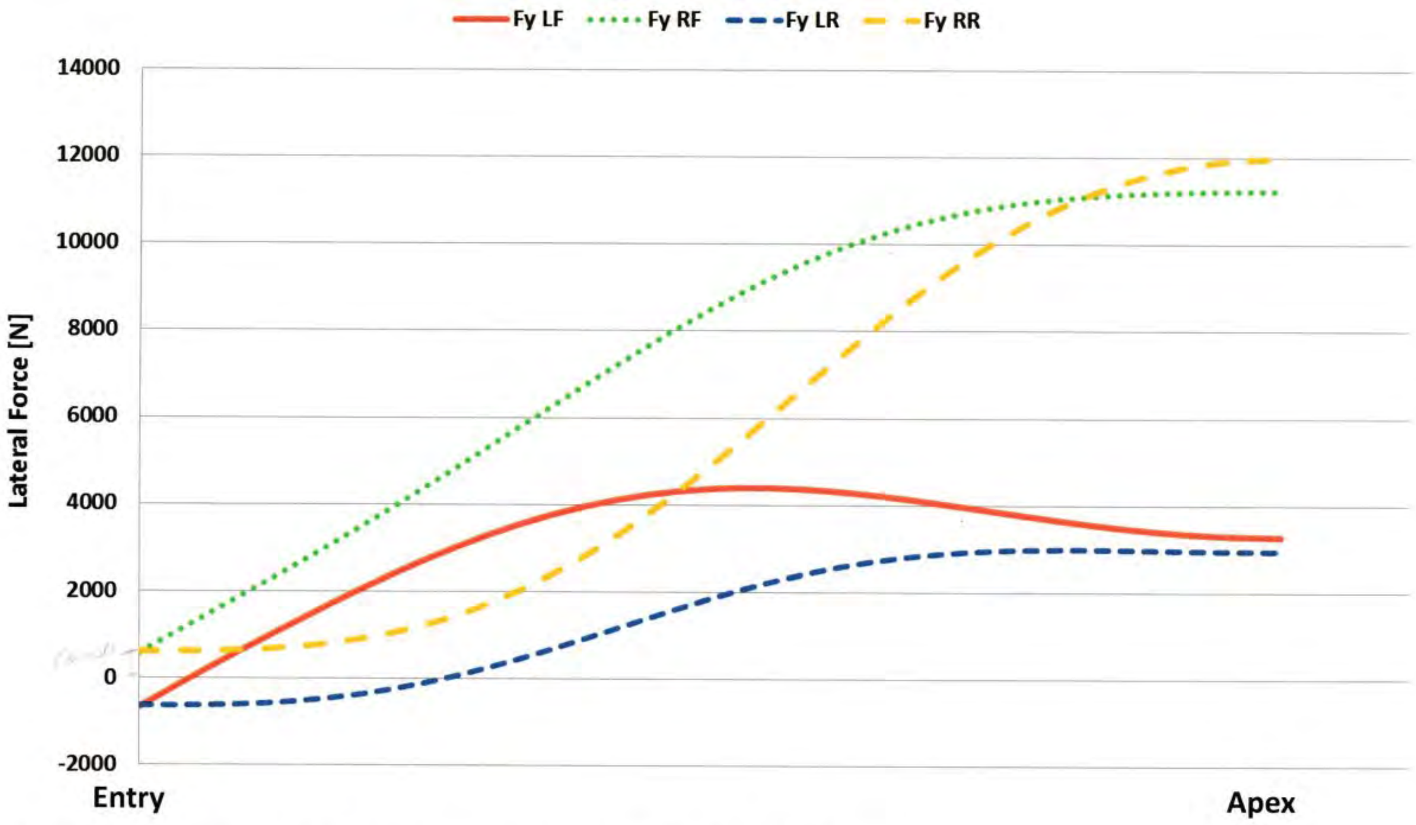
Lateral Forces



Note : Lateral Force from corner entry to apex (0N.m) – Left Turn.

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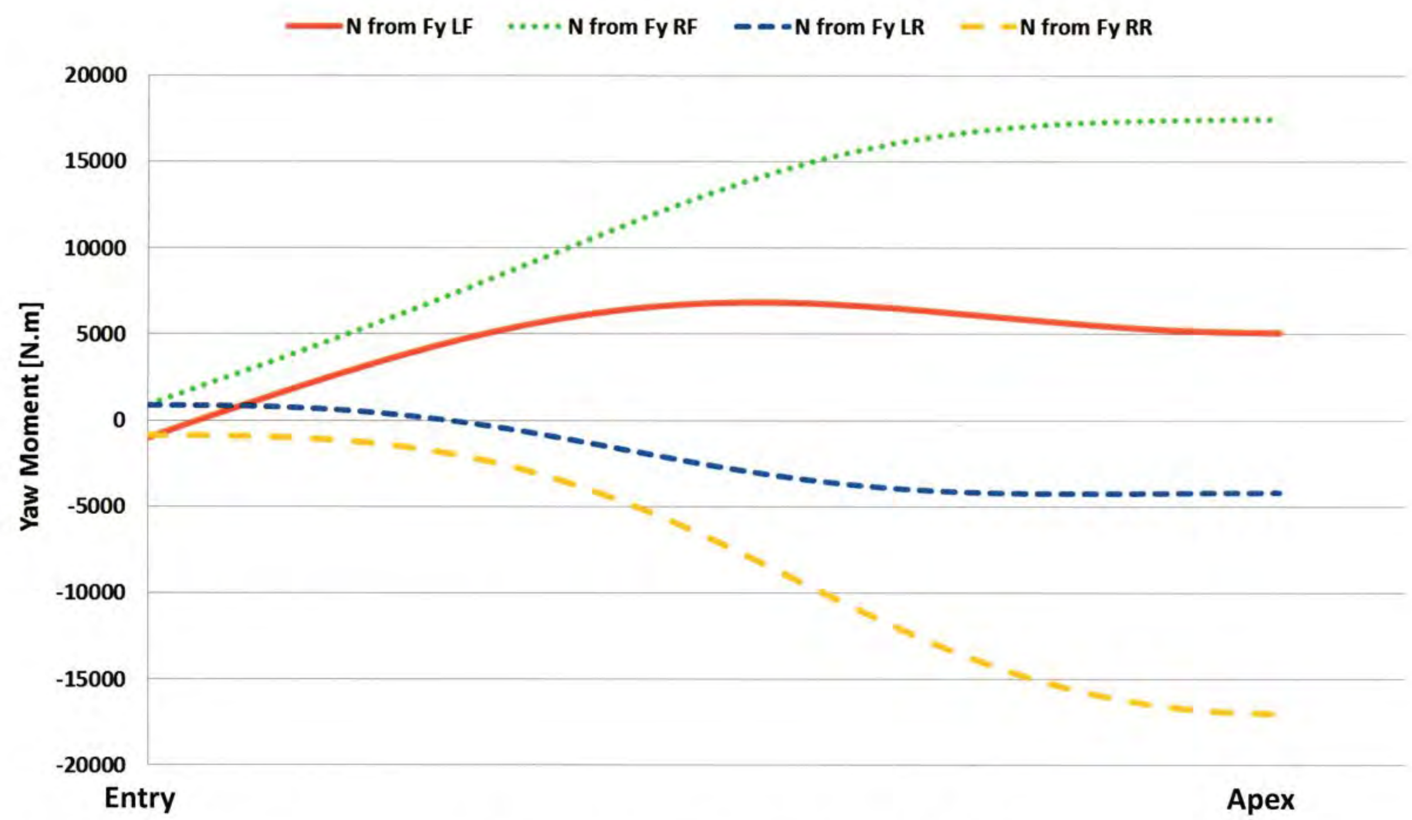
Yaw Moment from Lateral Forces



Note : Dynamic yaw moment from corner entry to apex (0N.m) – Left Turn.

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Discussion

- Please form 4 -5 Groups, I propose to mix up, between the universities.

- Discuss following Questions:
  - Other didactic approaches to introduced topics
  - Topics I missed generally (compared to overview sheet)
  - Topics we cancelled, because we don’t think, they are so important.

- Presentation and discussion of your results.

Literature

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